Problem Set: Sequences

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1) Show that the following sequences converge by the $\epsilon - K$ definition.

(i) $\lim_{n\to\infty} \frac{2n}{n+4\sqrt{n}} = 2$

(ii) $\lim_{n\to\infty} \frac{10^7}{n} = 0$

(iii) $\lim_{n\to\infty} \frac{n^2-1}{2n^2+3} = \frac{1}{2}$

(iv) $\lim_{n\to\infty} \frac{3n+1}{2n+3} = \frac{3}{2}$

2) Show that the sequence $x_n = \frac{1}{\ln(n+1)}$ converges to 0 using the $\epsilon - K$ definition. Also find the constant $K(\epsilon)$ when $\epsilon = \frac{1}{2}$ and $\epsilon = \frac{1}{10}$.

3) Discuss the convergence/divergence of the following sequences $(0 < a < 1)$ and $(b > 1)$.

(i) $x_n = \frac{n^2}{n+5}$

(ii) $x_n = \frac{n}{10^7}$

(iii) $x_n = \sqrt{n+1} - \sqrt{n}$

(iv) $x_n = \frac{(-1)^n}{n+1}$

(v) $x_n = \frac{1-2n}{1+2n}$

(vi) $x_n = \frac{1-5n^4}{n^4+8n^3}$

(vii) $x_n = \frac{\cos n}{n}$

(viii) $x_n = \frac{1}{3^n}$

(ix) $x_n = \frac{n^2}{e^n}$

(x) $x_n = a^n$

(xi) $x_n = \frac{n!}{n^n}$

(xii) $x_n = \frac{2^{3n}}{3^{2n}}$

(xiii) $x_n = \frac{n}{b^n}$

(xiv) $x_n = \frac{b^n}{n^2}$

(xv) $x_n = \frac{5^n}{n!}$

4) Show that sequence is monotone and bounded. Then find the limit.

(i) $x_1 = 1; x_{n+1} = \frac{x_n+1}{3}$

(ii) $x_1 = 2; x_{n+1} = \sqrt{2x_n + 1}$

5) Discuss whether the following sequences are Cauchy or not.

(i) $x_n = \frac{1}{n^2}$

(ii) $x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$

(iii) $x_n = \ln n^2$

(iv) $x_n = \sqrt{n}$

6) Show that the sequence $x_n = \frac{4-7n^6}{n^6+3}$ converges using the $\epsilon - K$ definition.

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7) Comment on the convergence of the sequence $x_n = \sin n$.

8) Does the recursively defined sequence $s_1 = 1; s_n = \frac{s_n+1}{5}$ converge? If so, find its limit.