भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology
Hyderabad

Teaching Learning Center
IIT Hyderabad

Problem Set: Series

J. Balasubramaniam†

1) Discuss the convergence/divergence of the following series:

   (i) $u_n = (\frac{1}{\sqrt{2}})^n$

   (ii) $u_n = (-1)^{n+1}\frac{3}{2^n}$

   (iii) $u_n = \sqrt{n+1} - \sqrt{n}$

   (iv) $u_n = e^{-2n}$

   (v) $u_n = \frac{2}{10^n}$

   (vi) $u_n = \frac{2^n - 1}{3^n}$

   (vii) $u_n = \frac{\cos n\pi}{5^n}$

   (viii) $u_n = \frac{n!}{1000^n}$

   (ix) $u_n = ln\frac{n}{n+1}$

   (x) $u_n = (\frac{e}{\pi})^n$

   (xi) $u_n = \frac{n!}{(2^n)^3}$

   (xii) $u_n = \frac{n^3}{2^n}$

   (xiii) $u_n = \frac{1}{n\sqrt{n+1}}$

   (xiv) $u_n = \frac{\sin^2 n}{n^2}$

   (xv) $u_n = \frac{e^{n\pi}}{\pi^{ne}}$

2) Show that the following series diverge:

   (i) $(-1)^n$ (ii) $\frac{n}{n+1}$ (iii) $\frac{n}{\sqrt{n^2+1}}$ (iv) $\cos\frac{n\pi}{2}$

3) Determine the conditional / absolute convergence of the following series:

   (i) $u_n = \frac{(-1)^n}{\ln n^2}$

   (ii) $u_n = \frac{(-2)^n}{n!}$

   (iii) $u_n = \frac{(-1)^n}{n \ln n^2}$

   (iv) $u_n = \frac{1}{\sqrt{n}} - \frac{1}{n}$

   (v) $u_n = \frac{(-3)^n}{n!}$

   (vi) $u_n = \frac{\cos n\pi}{\sqrt{n}}$

4) Let $p \geq 0$ and consider the series $\sum \frac{(-1)^{n-1}}{n^p}$. Determine for what values of p is the series conditionally / absolutely convergent.
5) Comment on the convergence of the following series:

   (i) $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$ (ii) $\sum_{n=1}^{\infty} \frac{e^{n\pi}}{\pi^{ne}}$ (iii) $\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^n$

6) Is the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \ln n}$ convergent? Is it absolutely convergent?
7) Give examples of sequences/series with the following properties:
   a) A sequence (x_n) such that $(|x_n|)$ converges but the original sequence (x_n) does not.
   b) Two divergent sequences $(x_n), (y_n)$ such that the sequence $(x_n y_n)$ converges.
   c) An unbounded sequence that has a convergent subsequence.
   d) Two divergent series $\sum x_n, \sum y_n$ such that the series $\sum x_n y_n$ converges.

† The author is with the Department of Mathematics, IIT Hyderabad 502285 India e-mail: jbala@iith.ac.in.

e) Two convergent series $\sum x_n, \sum y_n$ such that the series $\sum x_n y_n$ diverges.
f) A convergent series $\sum x_n$ such that the series $\sum x_n^2$ diverges.
g) A convergent series $\sum x_n$ such that the series $\sum \sqrt{x_n}$ diverges.