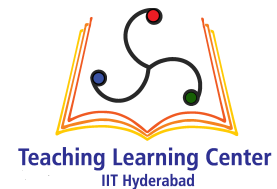




## Problem Set: Series



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1) Discuss the convergence/divergence of the following series:

(i)  $u_n = \left(\frac{1}{\sqrt{2}}\right)^n$

(vi)  $u_n = \frac{2^n - 1}{3^n}$

(xi)  $u_n = \frac{n!}{(2^n)^3}$

(ii)  $u_n = (-1)^{n+1} \frac{3}{2^n}$

(vii)  $u_n = \frac{\cos n\pi}{5^n}$

(xii)  $u_n = \frac{n^3}{2^n}$

(iii)  $u_n = \sqrt{n+1} - \sqrt{n}$

(viii)  $u_n = \frac{n!}{1000^n}$

(xiii)  $u_n = \frac{1}{n\sqrt{n+1}}$

(iv)  $u_n = e^{-2n}$

(ix)  $u_n = \ln \frac{n}{n+1}$

(xiv)  $u_n = \frac{\sin^2 n}{n^2}$

(v)  $u_n = \frac{2}{10^n}$

(x)  $u_n = \left(\frac{e}{\pi}\right)^n$

(xv)  $u_n = \frac{e^{n\pi}}{\pi^{ne}}$

2) Show that the following series diverge:

(i)  $(-1)^n$

(ii)  $\frac{n}{n+1}$

(iii)  $\frac{n}{\sqrt{n^2+1}}$

(iv)  $\cos \frac{n\pi}{2}$

3) Determine the conditional / absolute convergence of the following series:

(i)  $u_n = \frac{(-1)^n}{\ln n^2}$

(iii)  $u_n = \frac{(-1)^n}{n \ln n^2}$

(v)  $u_n = \frac{(-3)^n}{n!}$

(ii)  $u_n = \frac{(-2)^n}{n!}$

(iv)  $u_n = \frac{1}{\sqrt{n}} - \frac{1}{n}$

(vi)  $u_n = \frac{\cos n\pi}{\sqrt{n}}$

4) Let  $p \geq 0$  and consider the series  $\sum \frac{(-1)^{n-1}}{n^p}$ . Determine for what values of  $p$  is the series conditionally / absolutely convergent.

5) Comment on the convergence of the following series:

(i)  $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$

(ii)  $\sum_{n=1}^{\infty} \frac{e^{n\pi}}{\pi^{ne}}$

(iii)  $\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^n$

6) Is the series  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \ln n}$  convergent? Is it absolutely convergent?

7) Give examples of sequences/series with the following properties:

- A sequence  $(x_n)$  such that  $(|x_n|)$  converges but the original sequence  $(x_n)$  does not.
- Two divergent sequences  $(x_n), (y_n)$  such that the sequence  $(x_n y_n)$  converges.
- An unbounded sequence that has a convergent subsequence.
- Two divergent series  $\sum x_n, \sum y_n$  such that the series  $\sum x_n y_n$  converges.

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- e) Two convergent series  $\sum x_n, \sum y_n$  such that the series  $\sum x_n y_n$  diverges.
- f) A convergent series  $\sum x_n$  such that the series  $\sum x_n^2$  diverges.
- g) A convergent series  $\sum x_n$  such that the series  $\sum \sqrt{x_n}$  diverges.