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GATE Problems on Optimization

Teaching Learning Center
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1) A transportation problem for which the costs, origin and availabilities, destination and requirements are given as follows:

	D_1	D_2	D_3	
Q_1	2	1	2	40
Q_2	9	4	7	60
Q_3	1	2	9	10
	40	50	20	

Check whether the following basic feasible solution

$$x_{11}=20, x_{13}=20, x_{21}=10, x_{22}=50$$
$$x_{33}=10 \text{ and } x_{12}=x_{23}=x_{32}=x_{33}=0$$

is optimal. If not, find an optimal solution.

2) The objective function of the dual problem for the following primal linear programming problem:
Maximize $f=2x_1+x_2$
Subject to

$$x_1-2x_2\geqslant 2,$$
$$x_1+2x_2=8,$$
$$x_1-x_2\leqslant 11,$$

with $x_1\geqslant 0$ and x_2 unrestricted in sign, is given by

(A) minimize $z=2y_1-8y_2+11y_3$
(B) minimize $z=2y_1+8y_2+11y_3$
(C) minimize $z=2y_1-8y_2-11y_3$
(D) minimize $z=2y_1+8y_2-11y_3$

3) Solve the following linear programming problem using the Simplex method:
Minimize f $=-40x_1-100x_2$
Subject to
$10x_1+5x_2\leqslant 2500,$
$4x_1+10x_2\leqslant 2000,$
$2x_1+3x_2\leqslant 900,$
$x_1\geqslant 0, x_2\geqslant 0.$

4) Consider the primal problem (LP)
max $4x_1+3x_2$
subject to
$x_1+x_2\leqslant 8$
$2x_1+x_2\leqslant 10$
$x_1\geqslant 0,\ x_2\geqslant 10$
together with its dual (LD).Then

(A) (LP) and (LD) both are infeasible.
(B) (LP) and (LD) both are feasible.
(C) (LP) is feasible but (LD) is infeasible.
(D) (LP) is infeasible but (LD) is feasible.

5) Let **Z*** denote the optimal value of LPP
max Z $=4x_1+6x_2+2x_3$
such that
$3x_1+2x_2+x_3=12$
$x_1\geqslant 0,\ x_2\geqslant 0,\ x_3\geqslant 0.$
Then

(A) $10\leqslant Z^*\leqslant 20$
(B) $20<Z^*\leqslant 30$
(C) $30<Z^*\leqslant 40$
(D) $Z^*>40$

6) Let x be a non-optimal feasible solution of a linear programming maximization problem and y a dual feasible solution. Then

(A) The primal objective value at x is greater than the dual objective value at y.
(B) The primal objective value at x could equal the dual objective value at y.
(C) The primal objective value at x is less than the dual objective value at y.
(D) The dual could be unbounded.

7) Consider the Linear Program

$$\text{Max}\sum_{i=1}^{4} c_i x_i,$$

subject to

$$\sum_{i=1}^{4} a_i x_i \leqslant a_0,$$
$$0 \leqslant x_1, x_2, x_3, x_4 \leqslant 1.$$

where $a_i > 0, c_i > 0$ for $i = 1, 2, 3, 4$ and $a_0 > 0$

(i) Write the dual of this Linear Programming Problem.

(ii) Assuming

$$\frac{c_1}{a_1} \geqslant \frac{c_2}{a_2} \geqslant \frac{c_3}{a_3} \geqslant \frac{c_4}{a_4},$$
$$a_1 + a_2 \leqslant a_0, \text{ and } a_1 + a_2 + a_3 > a_0,$$

show that the feasible solution

$$x_1 = x_2 = 1, x_3 = \frac{a_0 - a_1 - a_2}{a_3}, x_4 = 0,$$

is an optimal solution.

8) Consider the optimal assignment problem, in which n persons $P_1, P_2, \ldots.., P_n$ are to be assigned n jobs $J_1, J_2, \ldots.., J_n$ and where the effectiveness rating of the person P_i for the job J_j is $a_{ij} > 0$. The objective is to find an assignment of persons to jobs, that is, a permutation $\sigma : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ which assigns person P_i to job $J_{\sigma(i)}$, so as to maximize the total effectiveness $\sum_{i=1}^{4} \alpha_{i\sigma(i)}$. Show that in any optimal assignment, at least one person is assigned a job at which he is best.

9) Suppose that the linear programming problem P : Min z $= c^1\ x\ s.t.\ A_x \geqslant b, x \geqslant 0$, where A is an $m \times n$ matrix, c an $n \times 1$ vector and b an $m \times 1$ vector, is being solved by the dual Simplex Algorithm. Then

(A) the value of the primal objective function increases at every iteration

(B) the algorithm will always terminate with an optimal solution for the dual

(C) the algorithm will always terminate with an optimal solution to the primal

(D) it is not always possible to obtain a starting basis for this algorithm

10) Consider the transportation problem given below. The bracketed elements in the table indicate a feasible solution and the elements on the left hand corner are the costs c_{ij}

				a_i
	2 (1)	5	1	1
	1	3 (1)	4 (1)	2
b_j	1	1	1	3

(A) this solution is a basic feasible solution

(B) this solution can be made basic feasible

(C) this is an optimal solution

(D) the problem does not have an optimal solution

11) Consider the linear programming formulation (P2) of optimally assigning n men to n jobs with respect to some costs $\{c_{ij}\}_{ij=1}^{n}$. Let A denote the coefficient matrix of the constraint set. Then,

(A) rank of A is 2n-1 and every basic feasible solution of P2 is integer valued.

(B) rank of A is 2n-1 and every basic feasible solution of P2 is not integer valued.

(C) rank of A is 2n and every basic feasible solution of P2 is integer valued.

(D) rank of A is 2n and every basic feasible solution of P2 is not integer valued.

12) Simplex tableau for phase I of the simplex algorithm for a linear programming problem is

given below (x_3,x_4,x_5 are artifical variables):

Basis	x_1	x_2	x_3	x_4	x_5	RHS
$Z_j - C_J$	0	0	-2	-2	0	0
x_1	1	0	$3/5$	$1/5$	0	2
x_2	0	1	$-2/5$	$1/5$	0	0
x_3	0	0	-1	-1	1	0

Choose the correct statement

(A) the tableau does not show the end of phase I, since the artifical variable x_5 is in the basis

(B) the tableau does show the end of phase I, since the value of the phase I objective function is zero

(C) the constraints for the original linear programming problem are not redundant

(D) the original linear programming problem does not have a feasible solution

13) Given below is the final tableau of a linear programming problem (x_4 and x_5 are slack variables):

Basis	x_1	x_2	x_3	x_4	x_5	RHS
$Z_j - C_J$	0	0	3	5	1	8
x_1	1	0	1	4	-1	2
x_2	0	1	2	-1	1	3

If the right hand side vector $\frac{1}{3}$ of the problem gets changed to $\frac{1+\theta}{3}$, then the current basic feasible solution is optimal for

(A) all $\theta \leqslant 2$

(B) all $\theta \geqslant -\frac{1}{4}$

(C) all $\theta \in \left[-\frac{1}{2}, 2\right]$

(D) no non-zero value of θ

14) Consider the Linear Programming Problem (LPP):
Maximize x_1,
subject to: $3x_1 + 4x_2 \leqslant 10$, $5x_1 - 2x_2 \geqslant -2$, $x_1 - 3x_2 \leqslant 3$, $x_1, x_2 \geqslant 0$.
The value of the LPP is

(A) $\frac{9}{5}$ (B) 2 (C) 3 (D) $\frac{10}{3}$

15) The unit cost c_{ij} of producing i at plant j is given by the matrix :

$$\begin{pmatrix} 14 & 12 & 16 \\ 21 & 9 & 17 \\ 9 & 7 & 5 \end{pmatrix}$$

The total cost of optimal assignment is

(A) 20

(C) 25

(B) 22

(D) 28

16) Consider the following primal Linear Programming Problem (LPP).
Maximize z$=3x_1+2x_2$
subject to $x_1 - x_2 \leqslant 1$

$x_1 + x_2 \geqslant 3$

$x_1, x_2 \geqslant 0$

The dual of this problem has

(A) infeasible optimal solution

(C) a unique optimal solution

(B) unbounded optimal objective value

(D) infinitely many optimal solutions

17) The cost matrix of a Transportation Problem is given by

6	4	1	5
8	9	2	7
4	3	6	2

The following values of the basic variables were obtained at the first iteration:
$x_{11} = 6,\ x_{12} = 8,\ x_{22} = 2,\ x_{23} = 14,\ x_{33} = 1,$ $x_{34} = 4$.
Then

(A) the current solution is optimal

(B) the current solution is nonoptimal and the entering and leaving variables are x_{31} and x_{33} respectively

(C) the current solution is nonoptimal and the entering and leaving variables are x_{21} and x_{12} respectively

(D) the current solution is nonoptimal and the entering and leaving variables are x_{14} and x_{12} respectively.

18) In a balanced transportation problem, if all the unit transportation costs c_{ij}, are decreased by a nonzero constant a, then in the optimal solution of the revised problem

(A) the values of the decision variables and the objective value remain unchanged

(B) the values of the decision variables change but the objective value remains unchanged

(C) the values of the decision variables remain unchanged but the objective value changes

(D) the value of the decision variables and the objective value change.

19) Consider the following Linear Programming Problem (LPP).
Maximize z$=3x_1+x_2$
subject to $x_1+2x_2\leqslant 5$
$x_1+x_2-x_3\leqslant 2$
$7x_1+3x_2-5x_3\leqslant 20$
$x_1,x_2,x_3\geqslant 0.$
The nature of the optimal solution to the problem is

(A) nondegenerate alternative optima
(B) degenerate alternative optima
(C) degenerate unique optimal
(D) nondegenerate unique optimal

20) For a linear programming primal maximization problem P with dual Q, which of the following statements is correct?

(A) The optimal values of P and Q exist and are the same

(B) Both optimal values exist and the optimal value of P is less than the optimal value of Q

(C) P will have an optimal solution, if and only if Q also has an optimal solution

(D) Both P and Q cannot be feasible

21) Let a convex set in 9-dimensional space be given by the solution set of the following system of linear inequalities

$$\sum_{i=1}^{3} \mathrm{x}_{ij} = 1, \quad i = 1,2,3$$
$$\sum_{j=1}^{3} \mathrm{x}_{ij} = 1, \quad j = 1,2,3$$
$$x_{ij} \geqslant 0, \quad i,j = 1,2,3$$

Then, the number of extreme points of this set is

(A) 3
(B) 4
(C) 9
(D) 6

22) Consider the linear programming problem

$$\begin{aligned} \text{Max } & c_1x_1+c_2x_2+c_3x_3 \\ \text{s.t } & x_1+x_2+x_3\leqslant 4 \\ & x_1\leqslant 2 \\ & x_3\leqslant 3 \\ & 3x_1+x_3\leqslant 7 \\ & x_1,x_2,x_3\geqslant 0. \end{aligned}$$

If (1,0,3) is an optimal solution, then

(A) $c_1\leqslant c_2\leqslant c_3$
(B) $c_3\leqslant c_1\leqslant c_2$
(C) $c_2\leqslant c_3\leqslant c_1$
(D) $c_2\leqslant c_1\leqslant c_3$

23) Let the convex set S be given by the solution set of the following system of linear inequalities in the sixteen variables $\{x_{ij} : i,j=1,......,4\}$:

$$\sum_{i=1}^{4} x_{ij} = 3, \quad i = 1, ..., 4$$

$$\sum_{j=1}^{4} x_{ij} = 3, \quad j = 1, ..., 4.$$

$$x_{ij} \geqslant 0, \quad i, j = 1, ..., 4.$$

Then, the dimension of S is equal to

(A) 4 (C) 8

(B) 9 (D) 12

Data for the following two questions:
Consider the Linear Programming Problem P:

$$\text{Max } c_1x_1 + c_2x_2 + ... + c_nx_n$$

$$\text{s.t.} \sum_{i=1}^{n} a_{ij}x_j \leqslant b_i, \quad i = 1, ..., m,$$

$$x_j \geqslant 0, \quad j = 1, ..., n,$$

with m constraints in n non-negative variables.

24) Let $x^* = (x_1^*, x_2^*, ..., x_n^*)$ be an optimal extreme point solution to P with $x_1^*, x_2^*, x_3^*, ..., x_n^* > 0$. Then, out of the m constraints $\sum_{j=1}^{n} a_{ij}x_j \leqslant b_i, \quad i = 1, ..., m$, the number of constraints not satisfied with equality at x^* is

(A) at most $m-4$ (C) equal to $m-3$

(B) at most $n-4$ (D) equal to $m-2$

25) Treat c_i's, a_{ij}'s as fixed and consider the problem P for different values of b_i's. Let P be unbounded for some set of parameters $b_1, b_2, ..., b_m$. Then

(A) $n > m$

(B) P is either unbounded or infeasible for every choice of b_i's

(C) $m > n$

(D) P has an optimal solution for some choice of b_i's.

26) Consider the linear programming problem,
max z $= c_1x_1 + c_2x_2, c_1, c_2 > 0$
Subject to $x_1 + x_2 \leqslant 3$
$2x_1 + 3x_2 \leqslant 4$

$x_1, x_2 \geqslant 0.$
Then,

(A) The primal has an optimal solution but the dual does not have an optimal solution.

(B) Both the primal and the dual have optimal solutions

(C) The dual has an optimal solution but the primal does not have an optimal solution

(D) Neither the primal nor the dual have optimal solutions.

27) For each $a \in \mathbb{R}$, consider the linear programming problem
Max z $= x_1 + 2x_2 + 3x_3 + 4x_4$
subject to

$$ax_1 + 2x_2 \leqslant 1$$

$$x_1 + ax_2 + 3x_4 \leqslant 2$$

$$x_1, x_2, x_3, x_4 \geqslant 0$$

Let S { $a \in \mathbb{R}$: the given LP problem has a basic feasible solution}. Then

(A) S $= \phi$ (C) S $= (0, \infty)$

(B) S = R (D) S $= (-\infty, 0)$

28) Consider the linear programming problem
Max z $= x_1 + 5x_2 + 3x_3$
subject to

$$2x_1 - 3x_2 + 5x_3 \leqslant 3$$

$$3x_1 + 2x_3 \leqslant 5$$

$$x_1, x_2, x_3 \geqslant 0.$$

Then the dual of this LP problem

(A) has a feasible solution but does not have a basic feasible soliution

(B) has a basic feasible solution

(C) has infinite number of feasible solutions

(D) has no feasible solution

29) Let $c_{ij} \geqslant 2$ be the cost of the $(i,j)^{th}$ cell of an assignment problem. If a new cost matrix is generated by the elements $c_{ij}' = \frac{1}{2}c_{ij} + 1$, then

(A) optimal assignment plan remains unchanged and cost of assignment decreases

(B) optimal assignment plan changes and cost of assignment decreases

(C) optimal assignment plan remains unchanged and cost of assignment increases

(D) optimal assignment plan changes and cost of assignment increases

30) Let a primal linear programming admit an optimal solution. Then the corresponding dual problem

(A) does not have a feasible solution

(B) has a feasible solution but does not have any optimal solution

(C) does not have a convex feasible region

(D) has an optimal solution

31) The cost matrix of a transportation problem is given by

1	2	3	4
4	3	2	0
0	2	2	1

The following are the values of variables in a feasible solution
$x_{12} = 6$, $x_{23} = 2$, $x_{24} = 6$, $x_{31} = 4$, $x_{33} = 6$
Then which of the following is correct??

(A) The solution is degenerate and basic

(B) The solution is non-degenerate and basic

(C) The solution is degenerate and non-basic

(D) The solution is non-degenerate and non-basic

32) The maximum value of z $= 3x_1 - x_2$ subject to $2x_1 - x_2 \leqslant 1$, $x_1 \leqslant 3$ and $x_1, x_2 \geqslant 0$ is

(A) 0 (B) 4 (C) 6 (D) 9

33) Consider the problem of maximizing z $= 2x_1 + 3x_2 - 4x_3 + x_4$ subject to

$$\begin{aligned} x_1 + x_2 + x_3 &= 2, \\ x_1 - x_2 + x_3 &= 2, \\ 2x_1 + 3x_2 + 2x_3 - x_4 &= 0, \\ x_1, x_2, x_3, x_4 &\geqslant 0. \end{aligned}$$

Then

(A) (1,0,1,4) is a basic feasible solution but (2,0,0,4) is not

(B) (1,0,1,4) is not a basic feasible solution but (2,0,0,4) is

(C) neither (1,0,1,4) nor (2,0,0,4) is a basic feasible solution

(D) both of (1,0,1,4) and (2,0,0,4) are basic feasible solutions

34) Which one of the following is TRUE??

(A) Every linear programming problem has a feasible solution.

(B) If a linear programming problem has an optimal solution then it is unique.

(C) The union of two convex sets is necessarily convex.

(D) Extreme points of the $x^2 + y^2 \leqslant 1$ are the point on the circle $x^2 + y^2 = 1$.

35) The dual of the linear programming problem Minimize $c^\tau x$ subject to $Ax \geqslant b$ and $x \geqslant 0$ is

(A) Maximize $b^\tau w$ subject to $A^\tau w \geqslant c$ and $w \geqslant 0$

(B) Maximize $b^\tau w$ subject to $A^\tau w \leqslant c$ and $w \geqslant 0$

(C) Maximize $b^\tau w$ subject to $A^\tau w \leqslant c$ and w is unrestricted

(D) Maximize $b^\tau w$ subject to $A^\tau w \geqslant c$ and w is unrestricted

36) The minimum value of
$z = 2x_1 - x_2 + x_3 - 5x_4 + 22x_5$ subject to

$$x_1 - 2x_4 + x_5 = 6$$
$$x_2 + x_4 - 4x_5 = 3$$
$$x_3 + 3x_4 + 2x_5 = 10$$
$$x_j \geqslant 0, \quad j = 1, 2, ..., 5$$

is

(A) 28 (B) 19 (C) 10 (D) 9

37) Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

5	23	14	8
10	25	1	23
35	16	15	12
16	23	11	7

is

(A) 29 (B) 52 (C) 26 (D) 44

38) The following table gives the cost matrix of a transportation problem

4	5	6
3	2	2
1	1	2

The basic feasible solution given by $x_{11} = 3$, $x_{13} = 1$, $x_{23} = 6$, $x_{31} = 2$, $x_{32} = 5$ is

(A) degenerate and optimal

(B) optimal but not degenerate

(C) degenerate but not optimal

(D) neither degenerate nor optimal

39) If z^* is the optimal value of the linear programming problem
Maximize $z = 5x_1 + 9x_2 + 4x_3$
subject to $x_1 + x_2 + x_3 = 5$
$4x_1 + 3x_2 + 2x_3 = 12$
$x_1, x_2, x_3 \geqslant 0$
then

(A) $0 \leqslant z^* < 10$ (C) $20 \leqslant z^* < 30$

(B) $10 \leqslant z^* < 20$ (D) $30 \leqslant z^* < 40$

40) The Linear Programming Problem:
Maximize $z = x_1 + x_2$
subject to

$$x_1 + 2x_2 \leqslant 20$$
$$x_1 + x_2 \leqslant 15$$
$$x_2 \leqslant 6$$
$$x_1, x_2 \geqslant 0$$

(A) has exactly one optimum solution

(B) has more than one optimum solution

(C) has unbounded solution

(D) has no solution

41) Consider the Primal Linear Programming Problem:

P: Maximize $z = c_1x_1 + c_2x_2 + ... + c_nx_n$
subject to
$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leqslant b_1$
$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leqslant b_2$
.
.
.
.
.
$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leqslant b_m$
$x_j \geqslant 0, \quad j = 1, ..., n.$

The Dual of P is

D: Minimize $z' = b_1w_1 + b_2w_2 + ... + b_mw_m$
subject to
$a_{11}w_1 + a_{12}w_2 + ... + a_{m1}w_m \leqslant c_1$
$a_{12}w_1 + a_{22}w_2 + ... + a_{m2}w_m \leqslant c_2$
.
.
.
.
.
$a_{1n}w_1 + a_{2n}w_2 + ... + a_{mn}w_m \leqslant c_n$
$w_i \geqslant 0, \quad i = 1, ..., m.$

Which of the following statements is FALSE?

(A) IF P has an optimal solution, then D also has an optimal solution

(B) The dual of the dual problem is a primal problem

(C) If P has an unbounded solution, then D has no feasible solution

(D) If P has no feasible solution, then D has a feasible solution

42) We have to assign four jobs I,II,III,IV to four workers A,B,C and D. The time taken by different workers (in hours) in completing different jobs is given below:
The optimal assignment is as follows:

		I	II	III	IV
	A	5	3	2	8
Workers	B	7	9	2	6
	C	6	4	5	7
	D	5	7	7	8

Job III to worker A; Job IV to worker B; Job II to worker C and Job I to worker D and hence the time taken by different workers in completing different jobs is now changed as: Then the minimum time (in hours) taken by

		I	II	III	IV
	A	5	3	2	5
Workers	B	7	9	2	3
	C	4	2	3	2
	D	5	7	7	5

the workers to complete all the jobs is

(A) 10 (B) 12 (C) 15 (D) 17

43) The following table shows the information on the availability of supply to each warehouse, the requirement of each market and unit of transportation cost (in rupees) from each warehouse to each market. The present transporta-

		Market M_1	M_2	M_3	M_4	Supply
	W_1	6	3	5	4	22
Warehouse	W_2	5	9	2	7	15
	W_3	5	7	8	6	8
Requirement		7	12	17	9	

tion schedule is as follows:
W_1 to M_2: 12 units; W_1 to M_3: 1 unit; W_1 to M_4: 9 units; W_2 to M_3: 15 units; W_3 to M_1: 7 units and W_3 to M_3: 1 unit. Then the minimum total transportation cost (in rupees) is

(A) 150 (B) 149 (C) 148 (D) 147

44) Consider the linear programming problem:

Maximize $x+\frac{3}{2}y$ subject to

$$2x+3y\leqslant 16,$$
$$x+4y\leqslant 18,$$
$$x\geqslant 0, y\geqslant 0.$$

If S denotes the set of all solutions of the above problem, then

(A) S is empty (C) S is a line segement

(B) S is a singleton (D) S has positive area

45) Consider the following linear programming problem:
Maximize $x+3y+6z-w$
subject to $5x+y+6z+7w\leqslant 20$,
$6x+2y+2z+9w\leqslant 40$,
$x\geqslant 0,\ y\geqslant 0,\ z\geqslant 0,\ w\geqslant 0$.
Then the optimal value is .....

46) LEt X be a convex region in the plane bounded by straight lines. Let X have 7 vertices. Suppose f$(x,y)=ax+by+c$ has maximum value M and minimum value N on X and N $< M$. Let S= {P: P is a vertex of X and N < f(P) < M}. If S has n elements, then which of the following statements is TRUE?

(A) n cannot be 5 (C) n cannot be 3

(B) n can be 2 (D) n can be 4

47) Consider the following linear programming problem:
Minimize x_1+x_2
Subject to:

$$2x_1+x_2\geqslant 8$$
$$2x_1+5x_2\geqslant 10$$
$$x_1,x_2\geqslant 0$$

The optimal value to this problem is ....

48) Consider the following linear programming problem:

Minimize: $x_1+x_2+2x_3$
Subject to

$$\begin{aligned} x_1+2x_2 &\geqslant 4 \\ x_2+7x_3 &\leqslant 5 \\ x_1-3x_2+5x_3 &= 6 \\ x_1,x_2\geqslant 0,\ x_3 &\text{ is unrestricted} \end{aligned}$$

The dual to this problem is:
Maximize: $4y_1+5y_2+6y_3$
Subject to

$$\begin{aligned} y_1+y_3 &\leqslant 1 \\ 2y_1+y_2-3y_3 &\leqslant 1 \\ 7y_2+5y_3 &= 2 \end{aligned}$$

and further subject to:

(A) $y_1\geqslant 0$, $y_2\leqslant 0$ and y_3 is unrestricted

(B) $y_1\geqslant 0$, $y_2\geqslant 0$ and y_3 is unrestricted

(C) $y_1\geqslant 0$, $y_3\leqslant 0$ and y_2 is unrestricted

(D) $y_3\geqslant 0$, $y_2\leqslant 0$ and y_1 is unrestricted

49) Consider the linear programming problem
Maximize $3x+9y$,
Subject to

$$\begin{aligned} 2y-x &\leqslant 2 \\ 3y-x &\geqslant 0 \\ 2x+3y &\leqslant 10 \\ x,y &\geqslant 0. \end{aligned}$$

Then the maximum value of the objective function is equal to ....

50) Minimize $w=x+2y$ subject to

$$\begin{aligned} 2x+y &\geqslant 3 \\ x+y &\geqslant 2 \\ x\geqslant 0, y &\geqslant 0 \end{aligned}$$

Then, the minimum value of w is equal to ....

51) Maximize $w=11x-z$ subject to

$$\begin{aligned} 10x+y-z &\leqslant 1 \\ 2x-2y+z &\leqslant 2 \\ x,y,z &\geqslant 0. \end{aligned}$$

Then, the maximum value of w is equal to ......

52) Consider the linear programming problem (LPP):
Maximize $4x_1+6x_2$
Subject to $x_1+x_2\leqslant 8$,
$2x_1+3x_2\geqslant 18$,
$x_1\geqslant 6$, x_2 is unrestricted in sign.
Then the LPP has

(A) no optimal solution

(B) only one basic feasible solution and that is optimal

(C) more than one basic feasible solution and a unique optimal solution

(D) infinitely many optimal solutions

53) For a linear programming problem (LPP) and its dual, which one of the following is NOT TRUE?

(A) The dual of the dual is primal

(B) If the primal LPP has an unbounded objective function, then the dual LPP is feasible

(C) If the primal LPP is infeasible, then the dual LPP must have unbounded objective function

(D) If the primal LPP has afinite optimal solution, then the dual LPP also has a finite optimal solution

54) Consider the following transportation problem. The entries inside the cells denote per unit cost of transportation from the origins to the destinations. The optimal cost of transportation equals ....

		Destination			
		1	2	3	Supply
	1	4	3	6	20
Origin	2	7	10	5	30
	3	8	9	7	50
	Demand	10	30	60	

55) Consider the linear programming problem (LPP):
Maximize kx_1+5x_2
Subject to $x_1+x_2\leqslant 1$,
$2x_1+3x_2\leqslant 1$,
$x_1,x_2\geqslant 0$.
If $x^* = (x_1^*,x_2^*)$ is an optimal solution of the above LPP with k=2, then the largest value of k (rounded to 2 decimal places) for which x^* remains optimal equals ....