# The Infinitesimal Dipole

G V S S Praneeth Varma and G V V Sharma*

Contents



***Abstract*—This manual provides an introduction to various parameters used in antenna design through the dipole.**

## 1 Electric and Magnetic Field

**Problem 1.** Show that

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \tag{1.1}$$

**Problem 2.** Let

$$A_x = 0, A_y = 0, A_z = \frac{\mu I_0 l}{4\pi r} e^{-\jmath kr} \tag{2.1}$$

Find $\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix}$.

**Problem 3.** Using

$$\mathbf{H_A} = \frac{1}{\mu} \nabla \times \mathbf{A}, \tag{3.1}$$

where the *curl* is defined as

$$\nabla \times \mathbf{A} = \begin{pmatrix} 0 & -\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} & \frac{1}{r}\left(\cot\theta + \frac{\partial}{\partial\theta}\right) \\ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} & 0 & -\frac{1}{r} - \frac{\partial}{\partial r} \\ -\frac{1}{r}\frac{\partial}{\partial\theta} & \frac{1}{r} + \frac{\partial}{\partial r} & 0 \end{pmatrix} \begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} \tag{3.2}$$

show that components of magnetic field are

$$\mathbf{H_A} = \begin{pmatrix} H_r \\ H_\theta \\ H_\phi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \jmath \frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{\jmath kr}\right] e^{-\jmath kr} \end{pmatrix} \tag{3.3}$$

**Problem 4.** Using

$$\mathbf{E_A} = -\jmath\omega\mathbf{A} - \jmath\frac{1}{\omega\mu\epsilon}\nabla\left(\nabla\cdot\mathbf{A}\right), \tag{4.1}$$

and Gradient and Divergence defined respectively as,

$$\nabla\mathbf{A} = \begin{pmatrix} \frac{\partial A_r}{\partial r} \\ \frac{1}{r}\frac{\partial A_\theta}{\partial\theta} \\ \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi} \end{pmatrix} \tag{4.2}$$

$$\nabla\cdot\mathbf{A} = \begin{pmatrix} \frac{2}{r} + \frac{\partial}{\partial r} \\ \frac{1}{r}\left(\cot\theta + \frac{\partial}{\partial\theta}\right) \\ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} \end{pmatrix}^T \begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} \tag{4.3}$$

*The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

show that components of electric field are

$$\mathbf{E_A} = \begin{pmatrix} E_r \\ E_\theta \\ E_\phi \end{pmatrix} = \begin{pmatrix} \eta \frac{I_0 l \cos\theta}{2\pi r^2}\left[1 + \frac{1}{\jmath kr}\right] e^{-\jmath kr} \\ \jmath\eta \frac{k I_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{\jmath kr} - \frac{1}{(kr)^2}\right] e^{-jkr} \\ 0 \end{pmatrix} \quad (4.4)$$

and find an expression for $\eta$.

**Problem 5.** For Near-field region ($rk << 1$), show that

$$\mathbf{E} \approx \begin{pmatrix} -\jmath\eta \frac{I_0 l \cos\theta}{2\pi k r^3} e^{-jkr} \\ -\jmath\eta \frac{k I_0 l \sin\theta}{4\pi k r^3} e^{-jkr} \\ 0 \end{pmatrix}, \quad \mathbf{H} \approx \begin{pmatrix} 0 \\ 0 \\ \frac{I_0 l \sin\theta}{4\pi r^2} e^{-\jmath kr} \end{pmatrix} \quad (5.1)$$

**Problem 6.** For Intermediate-field region ($rk > 1$), show that

$$\mathbf{E} \approx \begin{pmatrix} \eta \frac{I_0 l \cos\theta}{2\pi k r^2} e^{-\jmath kr} \\ \jmath\eta \frac{k I_0 l \sin\theta}{4\pi r} e^{-\jmath kr} \\ 0 \end{pmatrix}, \quad \mathbf{H} \approx \begin{pmatrix} 0 \\ 0 \\ \jmath \frac{k I_0 l \sin\theta}{4\pi r} e^{-\jmath kr} \end{pmatrix} \quad (6.1)$$

**Problem 7.** For Far-field region ($rk >> 1$), show that

$$\mathbf{E} \approx \begin{pmatrix} 0 \\ \jmath\eta \frac{k I_0 l \sin\theta}{4\pi r} e^{-\jmath kr} \\ 0 \end{pmatrix}, \quad \mathbf{H} \approx \begin{pmatrix} 0 \\ 0 \\ \jmath \frac{k I_0 l \sin\theta}{4\pi r} e^{-\jmath kr} \end{pmatrix} \quad (7.1)$$

## 2 Poynting Vector and Power

**Problem 8.** Show that

$$\mathbf{W} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^* = \begin{pmatrix} \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left[1 - \jmath\frac{1}{(kr)^3}\right] \\ \jmath\eta \frac{k|I_0 l|^2 \sin\theta\cos\theta}{16\pi^2 r^3}\left[1 + \frac{1}{(kr)^2}\right] \\ 0 \end{pmatrix} \quad (8.1)$$

**W** is known as the *Poynting* vector.

**Problem 9.** Let

$$d\mathbf{s} = \begin{pmatrix} r^2 \sin\theta \, d\theta d\phi \\ 0 \\ 0 \end{pmatrix}, 0 \le \theta \le \pi, 0 \le \phi \le 2\pi \quad (9.1)$$

Show that power in the radial direction

$$P = \oiint_S \mathbf{W} \cdot d\mathbf{s} = \frac{\eta\pi}{3}\left|\frac{I_0 l}{\lambda}\right|^2 \left[1 - \frac{\jmath}{kr^3}\right] \quad (9.2)$$

**Problem 10.** Find

$$P_{\text{rad}} = \Re\{P\} \quad (10.1)$$

$$2\omega\left(\tilde{W}_m - \tilde{W}_e\right) = \Im\{P\} \quad (10.2)$$

**Problem 11.** If

$$P_{\text{rad}} = \frac{1}{2}|I_0|^2 R_r, \quad (11.1)$$

find $R_r$, which is known as the *radiation resistance*.

## 3 Directivity

**Problem 12.** Find the *average power density*

$$\mathbf{W}_{\text{av}} = \frac{1}{2}\Re\{\mathbf{E} \times \mathbf{H}^*\} \quad (12.1)$$

and the *radiation intensity*

$$U = r^2 W_{\text{av}} \quad (12.2)$$

**Problem 13.** Show that

$$U_{\max} = \frac{\eta}{2}\left(\frac{k I_0 l}{4\pi}\right)^2 \quad (13.1)$$

**Problem 14.** Find the *directivity*

$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} \quad (14.1)$$

and the *maximum effective aperture*

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 \quad (14.2)$$

## 4 Radiation Pattern

**Problem 15.** Plot the polar graph for

$$U = \frac{\eta}{2}\left(\frac{k I_0 l}{4\pi}\right)^2 sin^2\theta \quad (15.1)$$

**Solution:** The desired graph is available in Fig. 15 through the following python code.

```
import numpy as np
import matplotlib.pyplot as plt

# infinitesimal dipole

c=3e8
f=2.4e9
lmbda=c/f
kl=2*np.pi/lmbda * lmbda/100
eta=120*np.pi
I= 1e-3
theta=np.linspace(0,2*np.pi,100)
U=(eta/2.0)*np.square(kl*I/(4*np.
    pi))* np.square(np.sin(theta))

plt.polar(theta,U)
#plt.savefig('../figs/inf_dipole.
    eps')
plt.show()
```

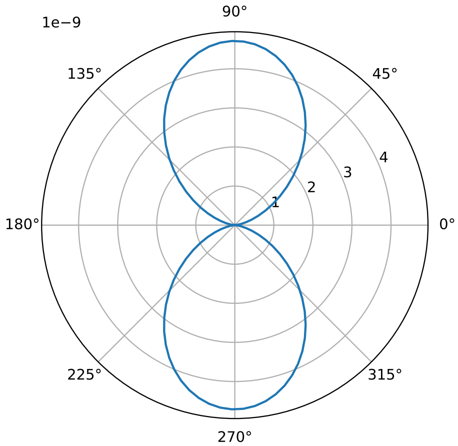


Fig. 15: Radiation Pattern

**Problem 16.** For a finite length dipole, the radiation intensity is given by

$$U = \eta \frac{|I_0|^2}{8\pi^2} \left[ \frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2 \tag{16.1}$$

Plot the polar graphs of the above equation for $l = \frac{\lambda}{4}, \frac{\lambda}{2}, \frac{3\lambda}{4}, \lambda, \frac{3\lambda}{2}$