Matrix Analysis Applications

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***Abstract*—This manual provides some examples of matrix analysis in research.**

Problem 1. Let

$$r = \sum_{j=1}^{2} h_j c_j \tag{1.1}$$

Express the above as a matrix equation. Note that r is a scalar.

Problem 2. Let

$$r_i = \sum_{j=1}^{2} h_{ij} c_j, \quad i = 1, 2. \tag{2.1}$$

Express the above as the matrix equation

$$\mathbf{r} = \mathbf{Hc} \tag{2.2}$$

List the entries of each matrix/vector in (2.2).

Problem 3. If

$$r_i = \sum_{j=1}^{N} h_{ij} c_j, \quad i = 1, 2 \dots M, \tag{3.1}$$

what is the dimension of the matrix $\mathbf{H}$ in the matrix equation?

Problem 4. Let

$$\mathbf{r}^t = \mathbf{h}^t \mathbf{C} \tag{4.1}$$

where $\mathbf{r}$ is $L \times 1$ vector and C is an $N \times L$ matrix. Find the least squares estimate for $\mathbf{h}$. What is the size of $\mathbf{h}$?

Problem 5. Now consider the matrix equation

$$\mathbf{R} = \mathbf{HC} \tag{5.1}$$

where $\mathbf{R}$ is $M \times L$, $\mathbf{H}$ is $M \times N$ and $\mathbf{C}$ is $N \times L$. Find the least squares estimate of $\mathbf{H}$.

Problem 6. Let

$$D = x_1^2 - x_2^2 \tag{6.1}$$

D can be expressed in quadratic form as $D = \mathbf{x}^t Q \mathbf{x}$, where $\mathbf{x} = (x_1, x_2)^t$. Find Q.

Problem 7. Find the determinant and eigenvalues of

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \tag{7.1}$$

Problem 8. Find the determinant and eigenvalues of $\mathbf{A} \otimes \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix. Comment.

Problem 9. Find the eigenvalues of $I - kA$, without explicitly calculating them. k is a constant.

Consider the matrix

$$\mathbf{S} = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \tag{9.1}$$

where $*$ represents the conjugate of a scalar and conjugate transpose of a vector.

Problem 10. Find SS^*. Comment.

Problem 11. Express

$$\begin{aligned} r_1 &= h_1 s_1 + h_2 s_2 \\ r_2 &= -h_1 s_2^* + h_2 s_1^* \end{aligned} \tag{11.1}$$

as a matrix equation.

Problem 12. Solve for s_1 and s_2 in (11.1) using matrices.

The problems in this chapter were framed using [1] and [2]. The primary reference for this manual is [3].

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