Geometry through Practice

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***Abstract*—This manual provides a unified approach for teaching primary and middle school mathematics by employing geometry for learning arithmetic. This is likely to speed up math learning besides helping the student apply mathematics in daily life. For best results, teachers and parents will have to create many examples similar to those available in the text. Also, students should be asked to draw all the figures themselves.**

Problem 1. The following figure is a *rectangle* with sides $AB = 6$cm and $BC = 8$cm. Draw the rectangle using a scale and protractor. Note that all angles in the rectangle are 90°

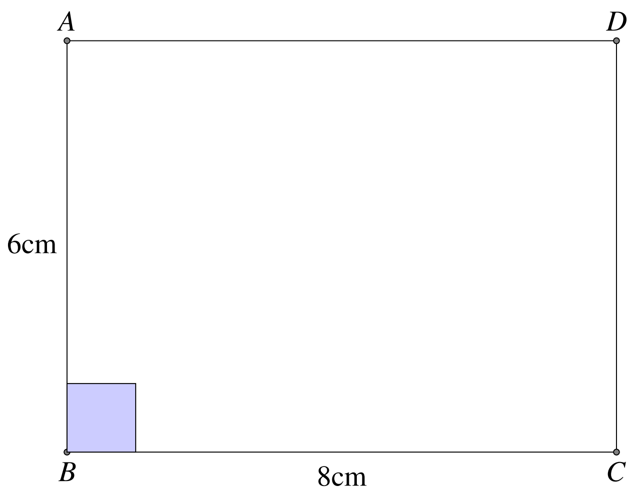

Fig. 1: Area of the rectange $= AB \times BC$.

Problem 2. Verify that $AC = 10$cm in Fig. 1.

Problem 3. The area of the rectangle $ABCD \triangleq AB \times BC$. Draw rectangles of different sizes and find their area.

Problem 4. Draw the line AC in Fig. 1 to get the *triangle ABC* as shown in Fig. 4.

Problem 5. Verify that the area of $\Delta ABC \triangleq \frac{1}{2} AB \times BC$. Draw various such triangles and find their area.

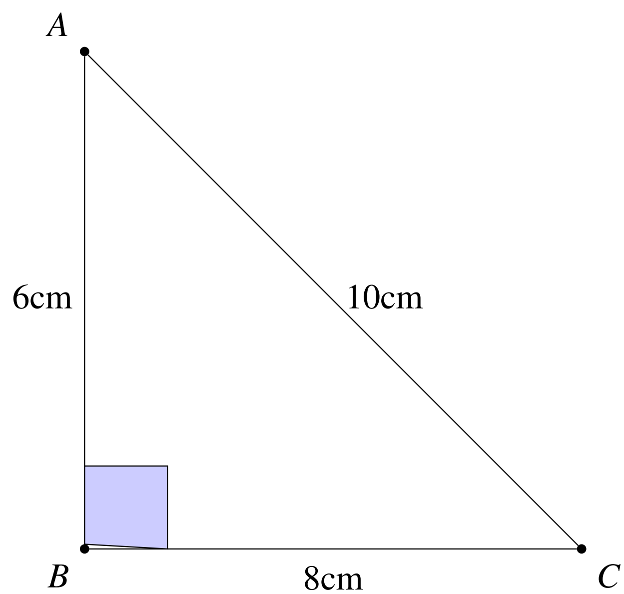

Fig. 4: Area of $\Delta ABC = \frac{1}{2} AB \times BC$.

Problem 6. The figure in Fig.6 is a *square* where all the sides are equal. Draw it using a scale and protractor. Note that all angles in the square are 90°. Find its area given by $AB \times AB = AB^2$.

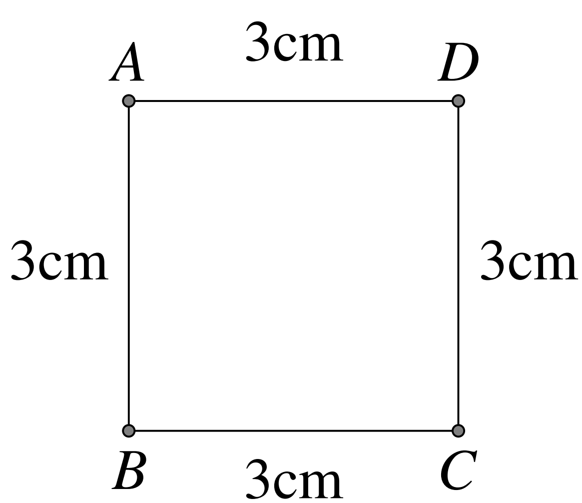

Fig. 6: Area of the square$= AB \times AB = AB^2$.

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Problem 7. In ΔABC in Fig. 4, verify that

$$AC^2 = AB^2 + BC^2 \tag{7.1}$$

Problem 8. In the Figure 8, $BCDF$ is a rectangle with CD = 6cm and DF = 8cm. Choose points A and E on the line DF such that $AE = BC = 8$cm. Join AB and CE. The figure $ABCE$ is known as a *parallelogram*, denoted as $||^{gm}$.

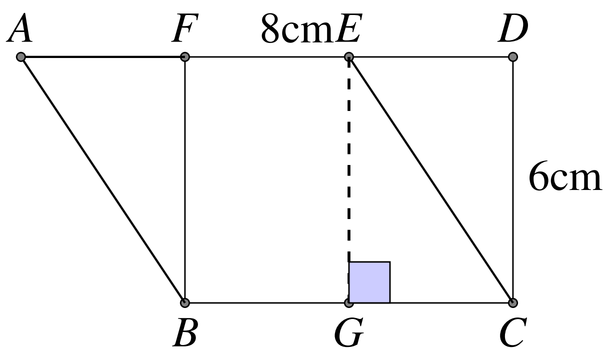

Fig. 8: Area of the parallelogram= $BC \times BF$.

Problem 9. Verify that the area

$$||^{gm}\ ABCE = \Delta ABF + \text{rect}BGEF + \Delta EGC \tag{9.1}$$
$$= \text{rect}ABCE = BC \times BF \tag{9.2}$$

Problem 10. Draw Figure 10 using a compass. This is known as a *circle* with *centre* O and *radius* r = 3cm. $AB = 2r$ is known as the *diameter* of the circle.

Problem 11. Draw the circle in Fig. 11 with AC as the diameter. Take any point B on the circle. Verify that $\angle ABC = 90^\circ$

1) Using a protractor.
2) Using (7.1).

Problem 12. Draw a line such that it touches the circle in Fig. 12 at the point P. Verify that the radius $OP \perp$ the tangent.

Problem 13. In ΔABC in Fig. 13, $BE \perp AC$ and $CF \perp AB$ are defined as the *altitudes*. Show that

$$\text{area of } \Delta ABC = \frac{1}{2} BE \times AC = \frac{1}{2} AB \times CF \tag{13.1}$$

Problem 14. BE and CF in Fig. 13 meet at O. Extend AO to meet BC at D. Verify that AD is also an altitude of the ΔABC.

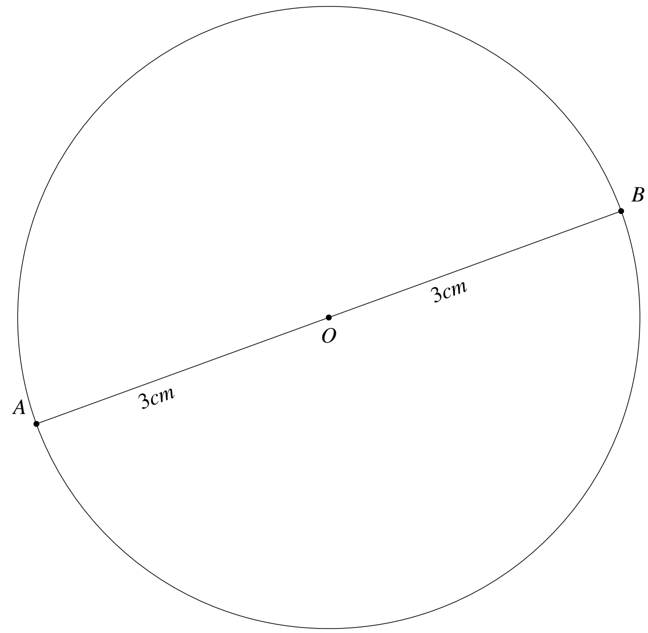

Fig. 10: Circle

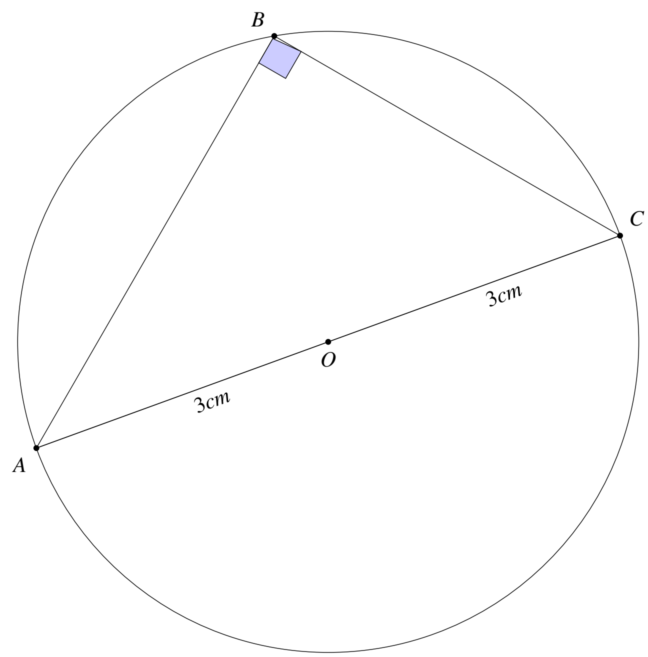

Fig. 11: Angle in a semi circle = 90°.

Problem 15. Draw the line BE such that it divides the side AC into two equal parts in ΔABC as shown in Fig. 15. BE is known as the *median*. CF is another median. BE and CF meet at O. Verify that

$$\frac{OE}{OB} = \frac{OF}{OC} = \frac{1}{2} \tag{15.1}$$

Problem 16. Extend the line AO in Fig. 15 to meet BC at D. Verify that AD is also a median.

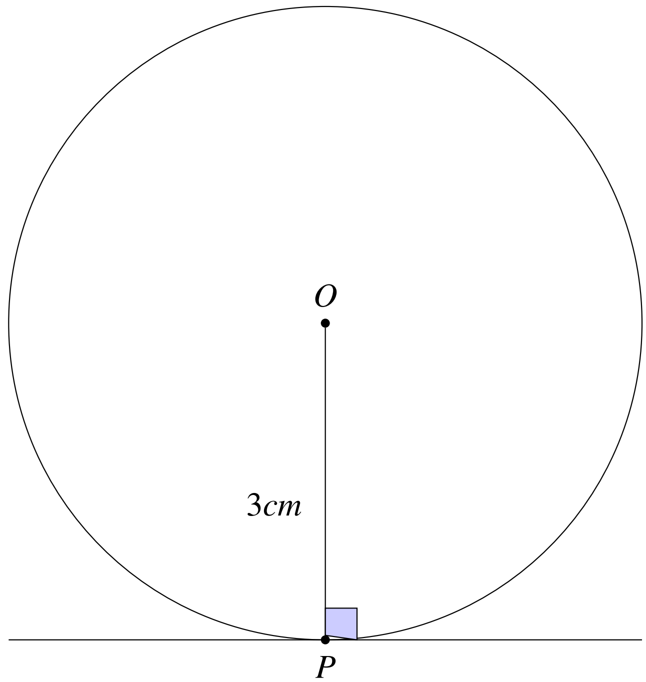

Fig. 12: Tangent to the circle.

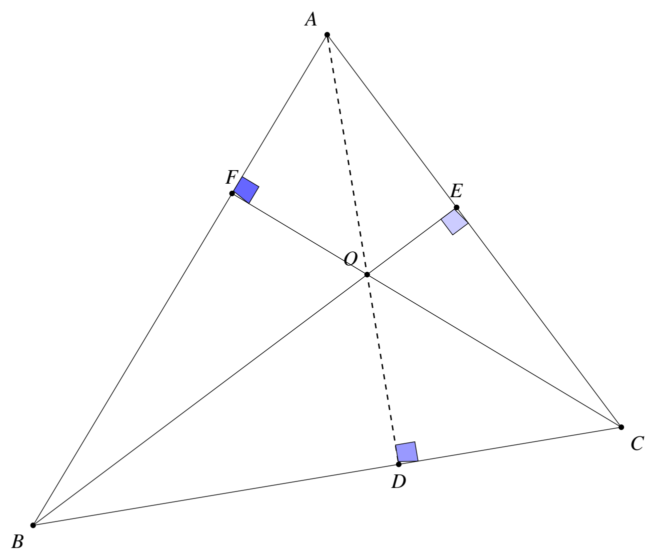

Fig. 13: Altitudes of a triangle meet in a point.

Problem 17. In ΔABC in Fig. 17, mark the mid points of BC, AC and AB respectively as D, E and F. Verify that

1) $\frac{EF}{BC} = \frac{DE}{AB} = \frac{DF}{AC} = \frac{1}{2}$
2) $\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta ABC} = \frac{1}{4}$
3) $\angle EDC = \angle DEF$
4) $EF \parallel BC = DE \parallel AB = DF \parallel AC$

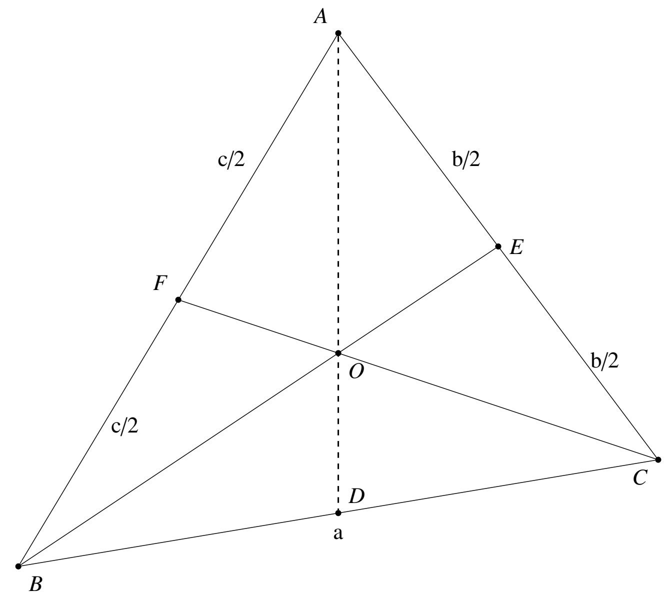

Fig. 15: Medians of a triangle meet in a point.

5) $\Delta DEF \sim \Delta ABC$.

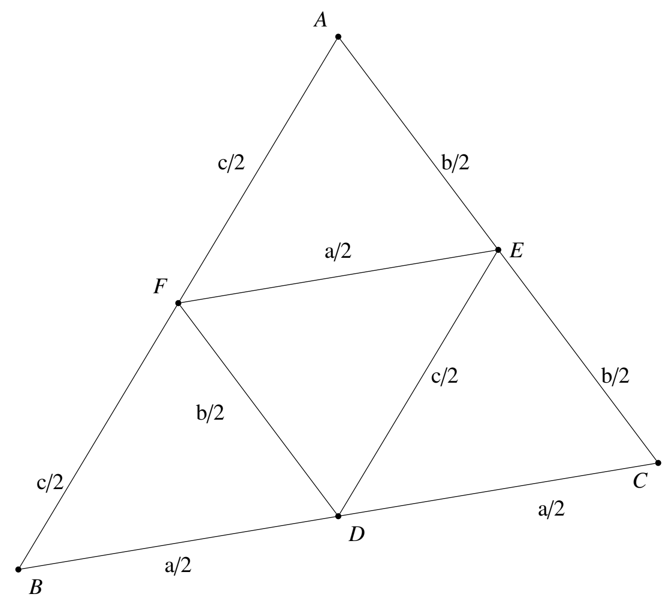

Fig. 17: Similar Triangles

Problem 18. Draw any circle with *diameter AB* as shown in Fig. 10 and verify that

$$\frac{\text{circumference}}{\text{diameter}} = \pi \approx \frac{22}{7} \tag{18.1}$$

Repeat this exercise for circles of different radii.

Problem 19. The area of a circle is given by πr^2. Calculate the areas of various circles of different radii.

Problem 20. Draw the *chords* AB and CD meeting at P as shown in Fig. 20. Join AC and BD and verify that $\Delta PAC \sim \Delta PDB$.

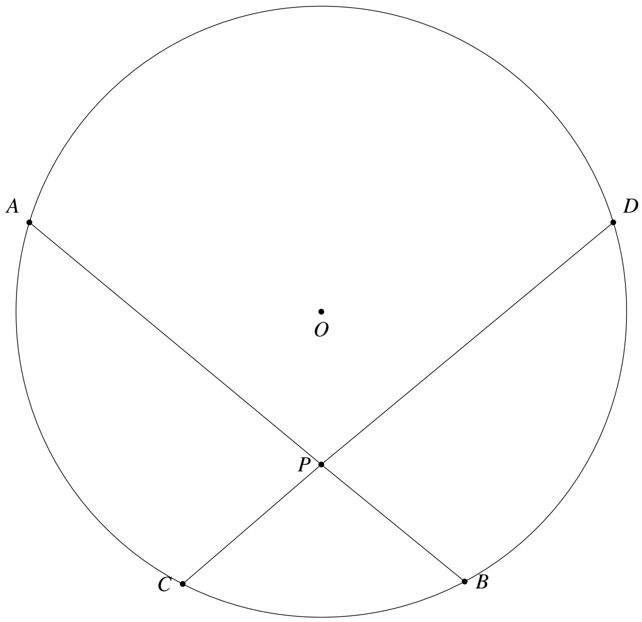

Fig. 20: $\Delta PAC \sim \Delta PDB$

Problem 21. Now join the lines OB and OC in Fig. 20 and verify that

$$\angle BOC = 2\angle BDC = 2\angle BAC \tag{21.1}$$

Problem 22. Draw the tangent through the point B and ΔABC as in Fig. 22. Verify that the marked angles are equal.

Problem 23. Draw the tangent PT to the circle as shown in Fig. 23 and a line PAB intersecting the circle at points A and B. Verify that

$$PA \times PB = PT^2 \tag{23.1}$$

Problem 24. In Fig. 24 draw tangents PA and PB to the circle where P is any point outside the circle. Verify that $PA = PB$.

Problem 25. In Fig. 25 draw the *angle bisectors* BD and CF such that

$$\angle ABD = \angle CBD \tag{25.1}$$

$$\angle ACF = \angle BCF \tag{25.2}$$

Verify that the line AO is also an angle bisector.

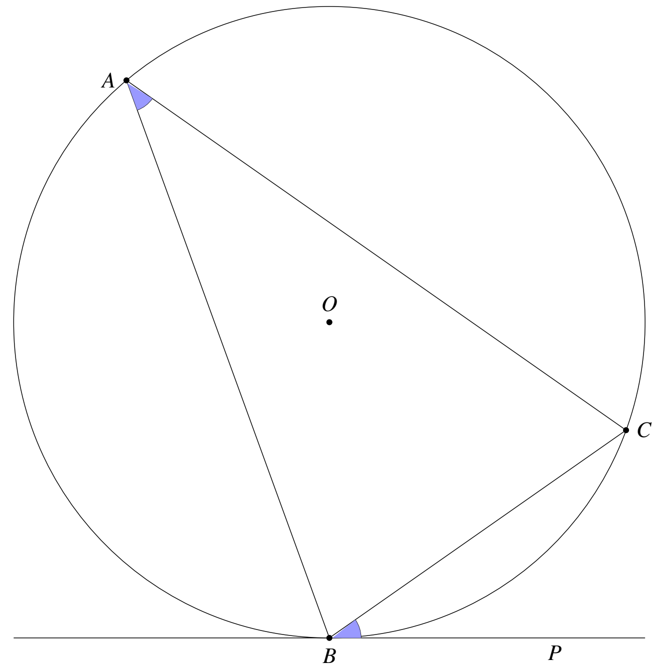

Fig. 22: Angles in the figure are equal.

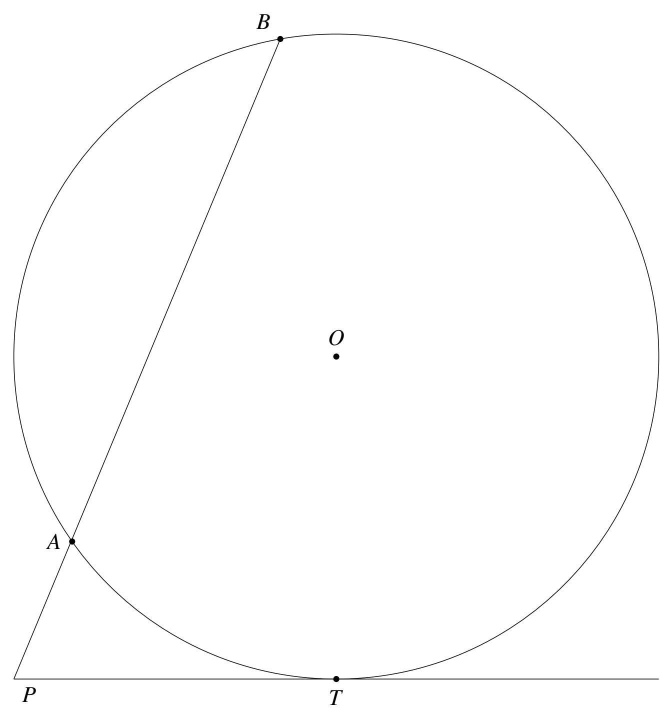

Fig. 23: $PA \times PB = PT^2$.

Problem 26. In Fig. 26 draw the *perpendicular bisectors* BD and CE meeting at the point O. Draw OD perpendicular to BC. Verify that $BD = DC$.

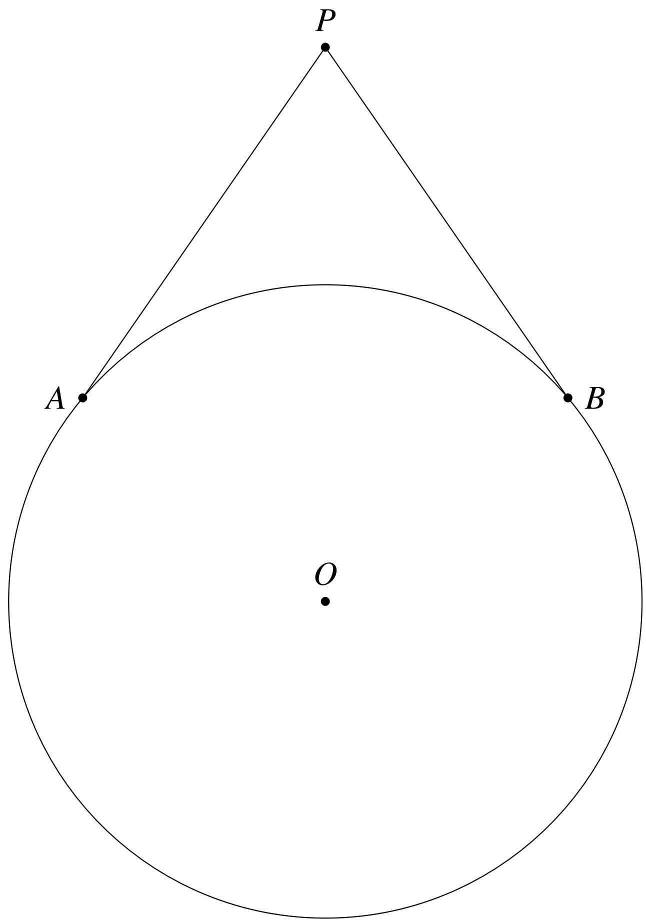

Fig. 24: $PA = PB$.

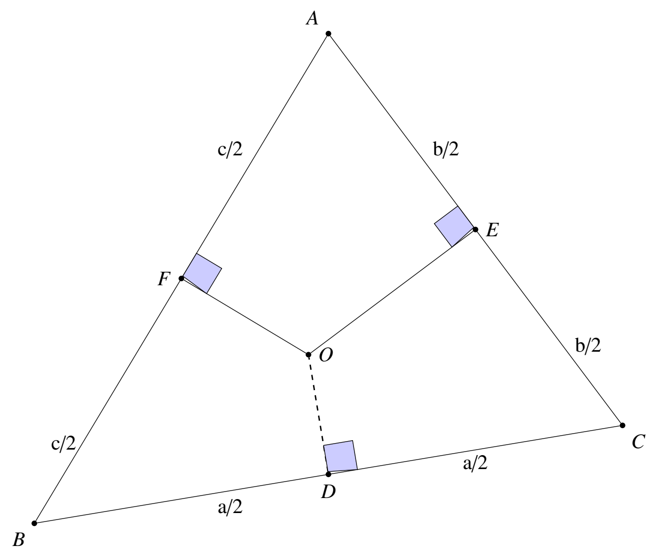

Fig. 26: Perpendicular bisectors meet at a point.

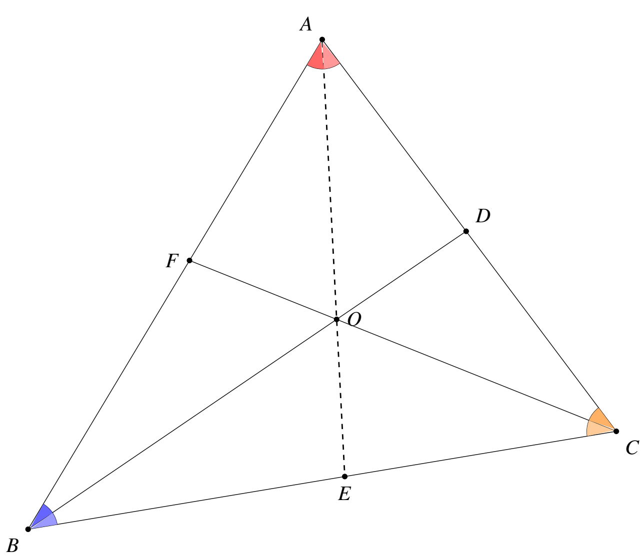

Fig. 25: Angles bisectors meet at a point.