Coordinate Geometry through LaTeX Tikz

Diparna Bose and G V V Sharma*

Contents

***Abstract*—This manual shows how to generate figures encountered in high school geometry using LaTeX Tikz. The process provides simple applications of coordinate geometry.**

1 Preliminaries

Problem 1. *Draw a circle of radius 1 unit with centre* $(0,0)$*. Mark the centre as O and A at* $45°$ *with the X-axis. Draw the radius OA and mark it as r.*

Solution: The following code results in Fig. 1.

```
\documentclass[10pt,a4paper]{
   article}
\usepackage{tikz}
\begin{document}
\providecommand{\brak}[1]{\
   ensuremath{\left(#1\right)}}

\begin{tikzpicture}
  [
     scale=2,
     >=stealth ,
     point/.style = {draw, circle ,
        fill = black, inner sep =
        0.5pt},
  ]
  \def\rad{1}
 \coordinate [point, label={below
    :        $O$ }] (O) at (0, 0);
  \draw (O) circle (\rad);
    \node (A) at +(45:{\rad}) [
       point,label = above right:$A
       $ ] {};
  \path
      (O)     edge   node[sloped,
         anchor=center, below, text
         width=0.5cm] { $r$}       (A)
         ;
\end{tikzpicture}

\end{document}
```

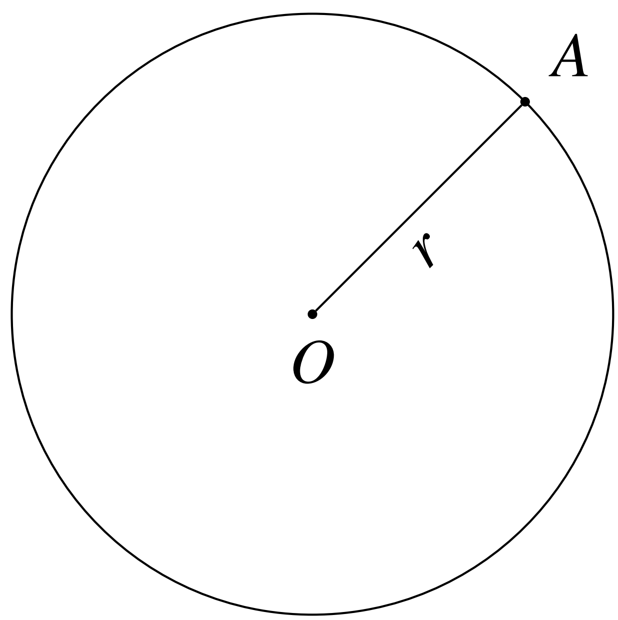

Fig. 1: Circle with radius $r = 1$.

Problem 2. *Note that the coordinates of A in Problem 1 are* $(\cos 45°, \sin 45°)$*. Use this information to draw the Fig. 1.*

*The authors are with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

Problem 3. *In Fig. 1,*

1) *Extend AO to the point B on the circle such that AB is a diameter.*
2) *Choose a point C on the circle using polar coordinates such that $\theta = 120°$.*
3) *Join AC and BC.*
4) *$\angle ACB = 90°$. Mark it as a right angle.*

Solution: The following code results in Fig. 3.

```
\documentclass[10pt,a4paper]{
   article}
\usepackage{tikz}
\usepackage{tkz-euclide} % loads
   TikZ and tkz-base
\usetkzobj{all}
\begin{document}
\providecommand{\brak}[1]{\
   ensuremath{\left(#1\right)}}

\begin{tikzpicture}
  [
    scale=2,
    >=stealth,
    point/.style = {draw, circle,
        fill = black, inner sep =
       0.5pt},
  ]
  \def\rad{1}
 \coordinate [point, label={below
    :      $O$ }] (O) at (0, 0);
    \node (A) at +(45:{\rad}) [
       point,label = above right:$A
       $ ] {};
    \node (B) at +(225:{\rad}) [
       point,label = below left:$B$
        ] {};
    \node (C) at +(120:{\rad}) [
       point,label = above left:$C$
        ] {};
  \path
     (B)     edge  node[sloped,
        anchor=east, below right,
        text width=0.5cm] { $d$}
            (A) ;
  \draw (A) -- (C);
  \draw (B) -- (C);
\draw
    (A) arc(45:225:\rad) -- cycle;
  \tkzMarkRightAngle[fill=blue!20,
     size=.2](A,C,B)

\end{tikzpicture}

\end{document}
```

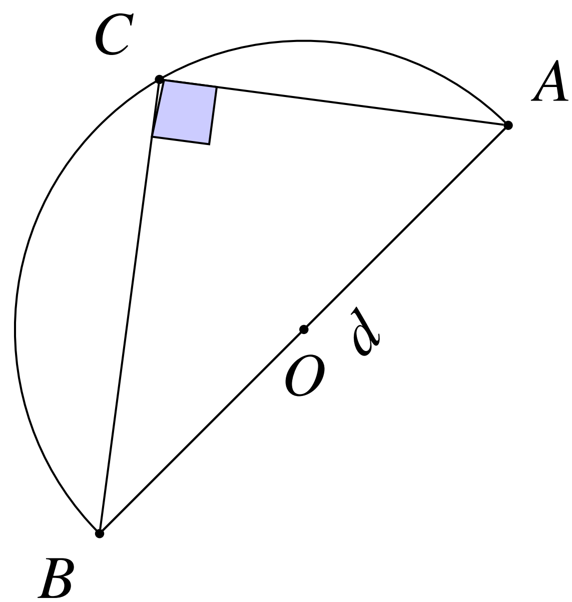

Fig. 3: Angle in the semi-circle is a right angle.

Problem 4. *Draw a $\triangle ABC$ with vertices*

$$A = \begin{pmatrix} -2 \\ -2 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, C = \begin{pmatrix} 4 \\ -1 \end{pmatrix}, \tag{1}$$

Solution: The following code results in Fig. 4 with the desired vertices.

```
\begin{tikzpicture}
  [
    scale=2,
    >=stealth,
    point/.style = {draw, circle,
        fill = black, inner sep =
       0.5pt},
  ]
\node (A) at (-2,-2) [point,label
   = below right:$A{(-2,-2)}$] {};
\node (B) at (1,3) [point,label =
   above left:$B{(1,3)}$] {};
\node (C) at (4,-1) [point,label =
    below right:$C{(4,-1)}$] {};
\draw (A) -- (C);
\draw (B) -- (C);
\draw (A) -- (B);
```

```
\end{tikzpicture}
```

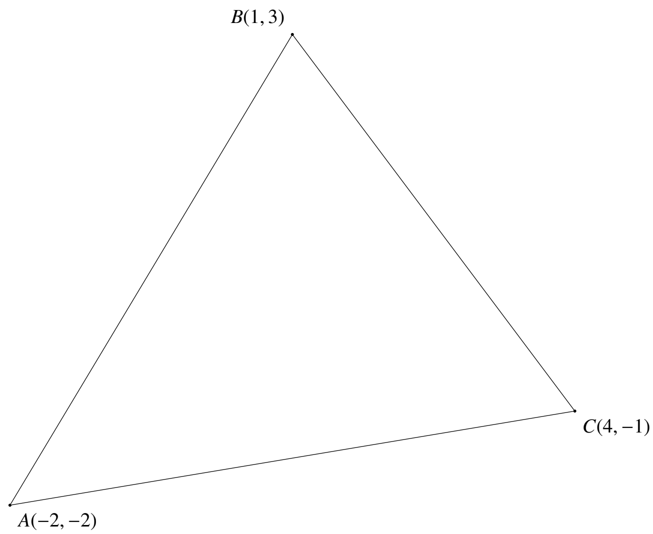

Fig. 4: Triangle.

2 Medians of a Triangle

Problem 5. *Find the coordinates of* D, E *and* F *of the mid points of* AB, BC *and* CA *respectively for the* ΔABC *in Problem 4.*

The coordinates of the mid points are given by

$$D = \frac{B+C}{2}, E = \frac{C+A}{2}, F = \frac{A+B}{2} \tag{2}$$

$$\implies D = \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix}, E = \begin{pmatrix} 1 \\ -\frac{3}{2} \end{pmatrix}, F = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \tag{3}$$

Problem 6. AD, BE *and* CF *are defined to be the medians of* ΔABC*. Draw them and verify that they meet at a point.*

Solution: The following code results in Fig. 6. Note that the medians meet at the *centroid*

$$G = \frac{A+B+C}{3} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{4}$$

```
\begin{tikzpicture}
  [
    scale=2,
    >=stealth,
    point/.style = {draw, circle,
        fill = black, inner sep =
      0.5pt},
  ]
\node (A) at (-2,-2) [point,label
  = below right:$A$] {};
\node (B) at (1,3) [point,label =
   above left:$B$] {};
\node (C) at (4,-1) [point,label =
    below left:$C$] {};
\draw (A) -- (C);
\draw (B) -- (C);
\draw (A) -- (B);

\node (D) at (2.5,1) [point,label
  = right:$D {(2.5,1)}$] {};
\node (E) at (1,-1.5) [point,label
   = below right:$E {(1,-1.5)}$]
   {};
\node (F) at (-0.5,0.5) [point,
   label = above left:$F
   {(-0.5,0.5)}$] {};
\draw (A) -- (D);
\draw (B) -- (E);
\draw (C) -- (F);

\node (G) at (1,0) [point,label =
   right:$G {(1,0)}$] {};


\end{tikzpicture}
```

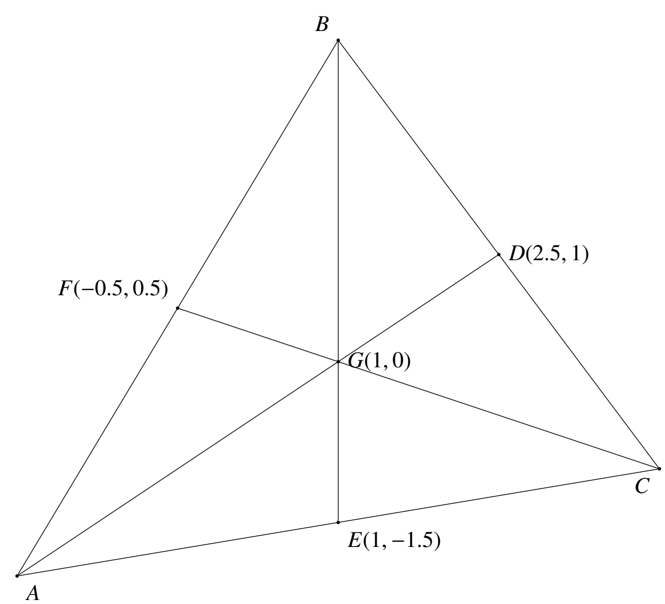

Fig. 6: Medians of ΔABC meet at G.

3 Altitudes of a Triangle

Definition 7. *In* ΔABC*, Let* P *be a point on* BC *such that* $AP \perp BC$*. Then* AP *is defined to be an* altitude *of* ΔABC*.*

Problem 8. *Find the equations of AB, BC and CA.*

Solution: Let

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}, C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \tag{5}$$

The equation of CA is given by

$$\frac{y - A_2}{x - A_1} = \frac{A_2 - C_2}{A_1 - C_1} \implies x - 6y - 10 = 0 \tag{6}$$

after some algebra. Similarly, the equations of AB and BC are

$$5x - 4y + 7 = 0 \tag{7}$$
$$4x + 3y - 5 = 0 \tag{8}$$

Problem 9. *Let the altitudes of the triangle be AP, BQ and CR. Find their equations.*

Solution: The equation for BQ is given by

$$y - B_2 = m_{BQ}(x - B_1) \tag{9}$$

where m_{BQ} is defined to be the slope of BQ. Since $BQ \perp CA$,

$$m_{BQ} m_{CA} = -1 \tag{10}$$

From (6), $m_{CA} = \frac{1}{6}$. Hence, from (10) and (9), the equation for BQ is

$$y - 3 = -6(x - 1) \tag{11}$$
$$\implies 6x + y - 9 = 0 \tag{12}$$

Similarly, the equations for AP and CR are

$$3x - 4y - 2 = 0 \tag{13}$$
$$4x + 5y + 9 = 0 \tag{14}$$

respectively.

Problem 10. *Find the coordinates of P, Q and R.*

Solution: $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$ is the intersection of BQ and CA whose equations are

$$6x + y - 9 = 0 \tag{15}$$
$$x - 6y - 10 = 0 \tag{16}$$

which result in the matrix equation

$$\begin{pmatrix} 6 & 1 \\ 1 & -6 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} 9 \\ 10 \end{pmatrix}. \tag{17}$$

Solving the above equation results in

$$Q = \begin{pmatrix} \frac{64}{37} \\ -\frac{51}{37} \end{pmatrix} \tag{18}$$

Similarly,

$$P = \begin{pmatrix} 2.32 \\ 1.24 \end{pmatrix}, R = \begin{pmatrix} 0.02941176 \\ 1.38235294 \end{pmatrix}, \tag{19}$$

Problem 11. *Draw AP, BQ and CR and verify that they meet at a point H.*

Solution: The following code results in Fig. 11. Note that the altitudes meet at *orthocentre H*.

```
\begin{tikzpicture}
[
 scale=2,
  >=stealth ,
  point/.style = {draw, circle ,
     fill = black , inner sep = 1pt
     },
]

\node (A) at (-2,-2) [point ,label
   = below left:$A$] {};
\node (B) at (1,3)[point ,label =
   above left:$B$] {};
\node (C) at (4,-1)[point ,label =
   below right:$C$] {};
\draw (A) -- (B) -- (C) -- (A);

\node (D) at (2.32 ,1.24) [point ,
   label = above right:$P
   {(2.32 ,1.24)}$] {};
\draw (A) -- (D);
\tkzMarkRightAngle[fill = blue!20,
   size=.2](A,D,C)

\node (E) at
   (1.72972973 ,-1.37837838)[point ,
   label = below:$Q {(1.73 ,-1.34)
   }$] {};
\draw (B) -- (E);
\tkzMarkRightAngle[fill = blue!40,
   size=.2](B,E,C)

\node (F) at
   (0.02941176 ,1.38235294)[point ,
   label = above left:$R
   {(0.03 ,1.4)}$] {};
\draw (C) -- (F);
```

```
\tkzMarkRightAngle[fill = blue!60,
    size=.2](A,F,C)

\node (H) at (1.40741,0.555556) [
    point,label = right:$H
    {(1.41,0.56)}$] {};

\end{tikzpicture}
```

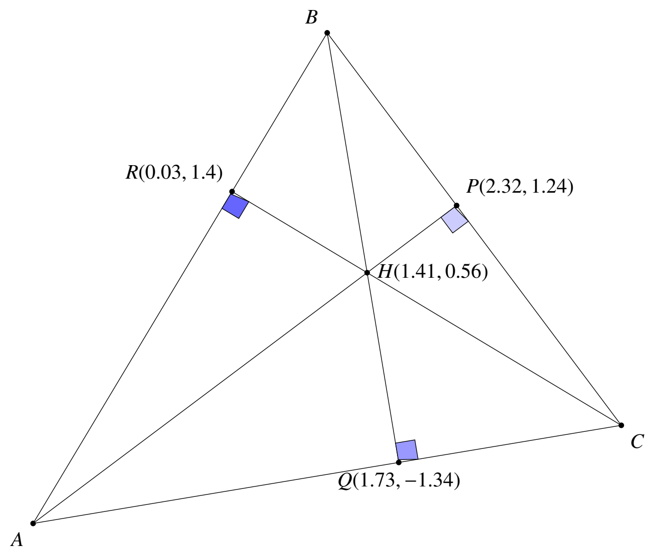

Fig. 11: Altitudes of ΔABC meet at H.

Problem 12. *Find the coordinates of H*

Solution: The coordinates of H are obtained by solving the equations for BQ and AP. The coordinates are available in Fig. 11.

4 Angle Bisectors of a Triangle

Definition 13. *In ΔABC, let U be a point on BC such that $\angle BAU = \angle CAU$. Then AU is known as the* angle bisector.

Problem 14. *Find the length of AB, BC and CA*

Solution: The length of CA is given by

$$CA = \sqrt{(C_1 - A_1)^2 + (C_2 - A_2)^2} = \sqrt{37}. \quad (20)$$

Similarly,

$$AB = \sqrt{34} \quad (21)$$
$$BC = 5 \quad (22)$$

Problem 15. *If AU, BV and CW are the angle bisectors, find the coordinates of U, V and W.*

Solution: Using the section formula,

$$W = \frac{AW.B + WB.A}{AW + WB} = \frac{\frac{AW}{WB}.B + A}{\frac{AW}{WB} + 1} \quad (23)$$
$$= \frac{\frac{CA}{BC}.B + A}{\frac{CA}{BC} + 1} \approx \begin{pmatrix} -0.35 \\ 0.75 \end{pmatrix} \quad (24)$$

since the angle bisector has the property that

$$\frac{AW}{WB} = \frac{CA}{AB} \quad (25)$$

Similarly,

$$U = \begin{pmatrix} 2.47 \\ 1.04 \end{pmatrix}, V = \begin{pmatrix} 1.23 \\ -1.46 \end{pmatrix} \quad (26)$$

Problem 16. *Draw AU, BV and CW and verify that they meet at a point I.*

Solution: The following code results in Fig. 16. Note that the angle bisectors meet at the *incentre* I.

```
\begin{tikzpicture}
[
  scale=2,
   >=stealth,
   point/.style = {draw, circle,
       fill = black, inner sep = 1pt
       },
]

\node (A) at (-2,-2)[point,label =
     below left:${A}$] {};
\node (B) at (1,3)[point,label =
    above left:${B}$] {};
\node (C) at (4,-1)[point,label =
    below right:${C}$] {};
\draw (A) -- (B) -- (C) -- (A);

\node (D) at (2.4682957,1.0422724)
    [point,label = above right:$U
    {(2.47,1.04)}$] {};
\draw (A) -- (D);
\node (E) at
    (1.23016035,-1.46163994)[point,
    label = below:$V {(1.23,-1.46)
    }$] {};
\draw (B) -- (E);

\node (F) at
    (-0.35345316,0.74424473)[point,
```

```
    label = left:$W {(-0.35,0.75)}$]
     {};
\draw (C) -- (F);

\node (I) at
    (1.14738665,0.14292163) [point,
    label =  right:$I {(1.15,0.14)
    }$] {};


\end{tikzpicture}
```

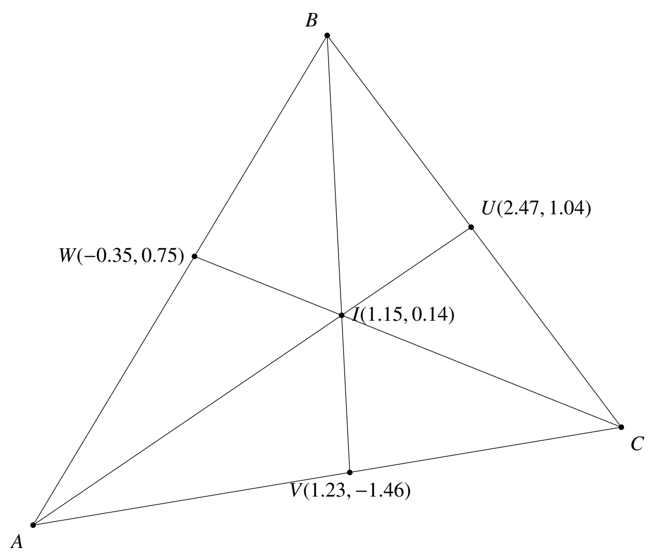

Fig. 16: Angle bisectors of ΔABC meet at I.

Problem 17. *Find the coordinates of I*

Solution:

$$I = \frac{BC.A + CA.B + AB.C}{AB + BC + CA} \tag{27}$$

$$= \begin{pmatrix} 1.15 \\ 0.14 \end{pmatrix} \tag{28}$$

5 Perpendicular Bisector

Problem 18. *Repeat the above exercises for the perpendicular bisectors of* ΔABC.