Finite Length Dipole

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***Abstract*—This manual derives the various parameters for a finite length dipole.**

Problem 1. The electric and magnetic field for the infinitesimal dipole in the far-field region ($rk >> 1$) is given by

$$\mathbf{E} \approx \begin{pmatrix} 0 \\ j\eta\frac{kI_0 l \sin\theta}{4\pi r}e^{-jkr} \\ 0 \end{pmatrix}, \quad \mathbf{H} \approx \begin{pmatrix} 0 \\ 0 \\ j\frac{kI_0 l \sin\theta}{4\pi r}e^{-jkr} \end{pmatrix} \tag{1.1}$$

Show that for length dz', the corresponding expressions are

$$\mathbf{E}' \approx \begin{pmatrix} 0 \\ j\eta\frac{kI_e dz' \sin\theta}{4\pi R}e^{-jkR} \\ 0 \end{pmatrix}, \quad \mathbf{H}' \approx \begin{pmatrix} 0 \\ 0 \\ j\frac{kI_e dz' \sin\theta}{4\pi R}e^{-jkR} \end{pmatrix} \tag{1.2}$$

where I_e is the current distribution, R is the distance from infinitesimal dipole at (0,0,z') on finite length dipole and the point of observation (x,y,z) as shown in Fig. 1

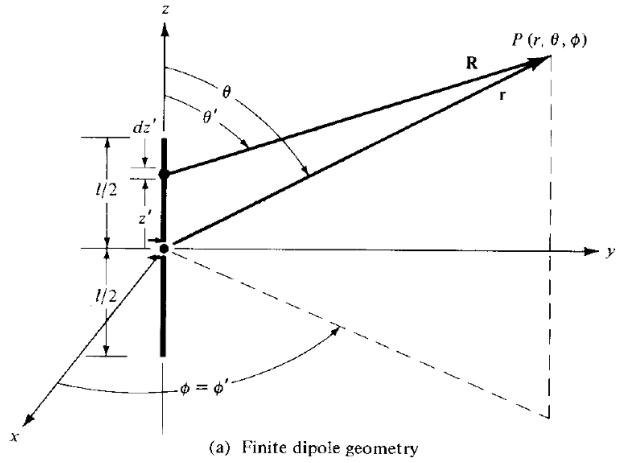

Fig. 1: Finite Length Dipole

Problem 2. Show that

$$R = \sqrt{r^2 + z'^2 - 2rz'\cos\theta} \tag{2.1}$$

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Problem 3. Show that

$$R \approx r - z'\cos\theta \tag{3.1}$$

when R is large.

Problem 4. Show that

$$\mathbf{E}' \approx \begin{pmatrix} 0 \\ j\eta\frac{kI_e dz' \sin\theta}{4\pi r}e^{-jkr}e^{jkz'\cos\theta} \\ 0 \end{pmatrix}, \tag{4.1}$$

$$\mathbf{H}' \approx \begin{pmatrix} 0 \\ 0 \\ j\frac{kI_e dz' \sin\theta}{4\pi r}e^{-jkr}e^{jkz'\cos\theta} \end{pmatrix} \tag{4.2}$$

Problem 5. Show that

$$\int e^{\alpha x}\sin(\beta x + \gamma)\,dx = \frac{e^{\alpha x}}{\alpha^2 + \beta^2}\left[\alpha\sin(\beta x + \gamma) - \beta\cos(\beta x + \gamma)\right] \tag{5.1}$$

Problem 6. Let

$$I_e = I_0 \sin\left[\frac{kl}{2}\Delta\left(\frac{2z'}{l}\right)\right] \tag{6.1}$$

where

$$\Delta(t) = \begin{cases} 1 - |t| & |t| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{6.2}$$

Show that

$$E_\theta = \int_{-l/2}^{l/2} E_\theta' dz' \tag{6.3}$$

$$\approx j\eta\frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right] \tag{6.4}$$

and

$$H_\phi \approx j\frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right] \tag{6.5}$$

Problem 7. Show that

$$U = \eta\frac{|I_0|^2}{8\pi^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2 \tag{7.1}$$