Recursive Least Squares Algorithm

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***Abstract*—This manual provides an introduction to the Adaptive Recursive Least Squares Algorithm.**

1 Problem Formulation

Consider the cost function

$$J(n) = \sum_{i=1}^{n} \beta(n,i)|e(n)|^2$$

where

$$e(n) = d(n) - W^T(n)X(n) \tag{1.1}$$

$$\text{and } 0 < \beta(n,i) \le 1 \tag{1.2}$$

Problem 1.1. Show that the optimal solution for

$$\min_W J(n) \tag{1.3}$$

is

$$W_*(n) = \phi^{-1}(n)z(n) \tag{1.4}$$

where

$$\phi(n) = \sum_{i=1}^{n} \lambda^{n-i} X(i)X^T(i) \tag{1.5}$$

$$z(n) = \sum_{i=1}^{n} \lambda^{n-i} X(i)d^T(i) \tag{1.6}$$

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Solution: The optimum value is obtained by solving the following equation

$$\frac{\partial J(n)}{\partial W(n)} = 0 \tag{1.7}$$

resulting in

$$\sum_{i=1}^{n} \lambda^{n-i}\Big[0 - X(i)d^T(i) - X^T(i)d(i) + 2W(n))X(i)X^T(i)\Big] = 0 \tag{1.8}$$

which can be expressed as

$$\left[\sum_{i=1}^{n} \lambda^{n-i} X(i)X^T(i)\right] W(n) = \sum_{i=1}^{n} \lambda^{n-i} X(i)d^T(i) \tag{1.9}$$

$$\implies \phi(n)W(n) = z(n) \tag{1.10}$$

Problem 1.2. Show that

$$\phi(n) = \lambda\phi(n-1) + X(n)X^T(n) \tag{1.11}$$

$$z(n) = \lambda z(n-1) + X(n)X^T(n) \tag{1.12}$$

2 Update Equations

Problem 2.1. If

$$A = B^{-1} + CD^{-1}C^T, \tag{2.1}$$

verify that

$$A^{-1} = B - BC(D + C^T BC)^{-1}C^T B \tag{2.2}$$

through a python script.

Problem 2.2. Using (2.2) and (1.11), show that

$$P(n) = \lambda^{-1}\left[I - \frac{\lambda^{-1}P(n-1)X(n)X^T(n)}{1 + \lambda^{-1}X^T(n)P(n-1)X(n)}\right]P(n-1) \tag{2.3}$$

where

$$P(n) = \phi^{-1}(n) \tag{2.4}$$

Problem 2.3. Show that

$$W(n) = W(n-1) + P(n)X(n)e(n) \tag{2.5}$$

3 RLS Algorithm

Problem 3.1. Obtain an algorithm for getting $e(n)$ from $d(n)$.

Solution:

1) Initialize the algorithm by setting $P(0) = \delta^{-1}I$, where δ is a small positive constant and $W^T(0) = 0$.
2) For $n = 1, 2, 3, \ldots$, compute the following

$$e(n) = d(n) - X^T(n)W(n-1) \tag{3.1}$$

$$W(n) = W(n-1) + P(n)X(n)e(n) \tag{3.2}$$

$$P(n) = \lambda^{-1}\left[I - \frac{\lambda^{-1}P(n-1)X(n)X^T(n)}{1 + \lambda^{-1}X^T(n)P(n-1)X(n)}\right]P(n-1) \tag{3.3}$$

Problem 3.2. Download the **RLS_NC_SPEECH.py** file from this link and execute it. Comapre the output with the LMS output.

4 Convergence of the RLS Algorithm

Problem 4.1. Show that the RLS algorithm converges in the mean as well as the mean square sense.