Transportation Problem

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Contents

***Abstract*—This manual explains the Northwest corner cell method, Modi Method, and using cvxopt for solving Transportation problems through examples.**

1 Northwest Corner Cell Method

Problem 1.1. Find the initial basic feasible solution of the following transportation problem.

S/D	D_1	D_2	D_3	D_4	SUPPLY
S_1	3	1	7	4	250
S_2	2	6	5	9	350
S_3	8	3	3	2	400
DEMAND	200	300	350	150	

TABLE 1.1.1

Solution:
Let the given cost matrix is

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{bmatrix} = \begin{bmatrix} 3 & 1 & 7 & 4 \\ 2 & 6 & 5 & 9 \\ 8 & 3 & 3 & 2 \end{bmatrix}$$

and the corresponding allocated supply/demand value for each cost cell is

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix}$$

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Then the solution is

$$f = \sum_{i=1}^{3} \sum_{j=1}^{4} x_{ij} C_{ij} \tag{1.1}$$

The basic feasible solution can be obtained by using Northwest corner cell method. The procedure is as follows

1. Select the Northwest corner cell from the TABLE 1.1.1. i.e. Element C_{11}
2. Get the supply, demand values corresponding to C_{11}.
3. Allocate the min{supply,demand} to the element C_{11}. Subtract the allocated value from both supply,demand values of element C_{11}. The allocated value(x_{11})=200. Then the table looks like TABLE 1.1.2

S/D	1	2	3	4	SUPPLY
1	3	1	7	4	50
2	2	6	5	9	350
3	8	3	3	2	400
DEMAND	0	300	350	150	

TABLE 1.1.2

4. The column or row corresponding to zero value of demand or supply will not be considered for allocation. i.e column corresponding to C_{11}.
5. Now again select the Northwest corner cell from the TABLE 1.1.2. i.e. Element C_{12}.
6. Allocate the min{supply,demand} to the element C_{12}. Subtract the allocated value from both supply,demand values of element C_{12}. The allocated value(x_{12})=50. Then the table looks like TABLE 1.1.3
7. Row corresponding to C_{12} is not considered for allocation.

   By repeating 1,2,3 steps one by one, the results are

8. The allocated value(x_{22})=250. Then the table looks like Table.1.1.4

S/D	1	2	3	4	SUPPLY
1	3	1	7	4	0
2	2	6	5	9	350
3	8	3	3	2	400
DEMAND	0	250	350	150	

TABLE 1.1.3

S/D	1	2	3	4	SUPPLY
1	3	1	7	4	0
2	2	6	5	9	100
3	8	3	3	2	400
DEMAND	0	0	350	150	

TABLE 1.1.4

9. The allocated value(x_{23})=100. Then the table looks like Table.1.1.5

S/D	1	2	3	4	SUPPLY
1	3	1	7	4	0
2	2	6	5	9	0
3	8	3	3	2	400
DEMAND	0	0	250	150	

TABLE 1.1.5

10. The allocated value(x_{33})=250. Then the table looks like Table.1.1.6

S/D	1	2	3	4	SUPPLY
1	3	1	7	4	0
2	2	6	5	9	0
3	8	3	3	2	150
DEMAND	0	0	0	150	

TABLE 1.1.6

11. The allocated value(x_{34})=150. Then the table looks like Table.1.1.7

S/D	1	2	3	4	SUPPLY
1	3	1	7	4	0
2	2	6	5	9	0
3	8	3	3	2	0
DEMAND	0	0	0	0	

TABLE 1.1.7

12. Remaining all other allocated values are zero.
13. Hence, the basic feasible solution is obtained. Then the total cost for the given transportation problem by using (1.1) = 3700 rupees.

2 UV or MODI Method

To optimize the given feasible solution, MODI method is used.

1. Arrange the table as like TABLE 2.0.8

	v_1	v_2	v_3	v_4
u_1	3	1	7	4
u_2	2	6	5	9
u_3	8	3	3	2

TABLE 2.0.8

and

$$X = \begin{bmatrix} 200 & 50 & 0 & 0 \\ 0 & 250 & 100 & 0 \\ 0 & 0 & 250 & 150 \end{bmatrix}$$

Now we have to find out values of $u_1, u_2, u_3, v_1, v_2, v_3, v_4$.

2. Always assume u_1=0. Then to find other values use (2.1) for allocated cells only.

$$u_i + v_j = C_{ij} \tag{2.1}$$

3. After finding all values of u and v, the table looks like Table.2.0.9

	v_1=3	v_2=1	v_3=0	v_4=-1
u_1=0	3	1	7	4
u_2=5	2	6	5	9
u_3=3	8	3	3	2

TABLE 2.0.9

4. Compute the penalties for the unallocated cells using (2.2).

$$P_{ij} = u_i + v_j - C_{ij} \tag{2.2}$$

5. Penalties are $P_{13} = -7, P_{14} = -5, P_{21} = 6, P_{24} = -5, P_{31} = -2, P_{32} = 1$
6. If we get all penalties as zero or less than zero then the given solution is optimal. If we get any penalty as positive, we need to proceed the problem to get optimum value.
7. Select the unallocated cell, which has maximum positive penalty. i.e. C_{21}
8. Draw a closed loop consisting only horizontal and vertical lines passing through some allocated cells only. i.e. $C_{21}C_{22}C_{12}C_{11}$.

9. Give the (+) sign to the starting cell in loop. Assign alternative signs to the other cells.
10. Select the least allocated value from the (-) signed cells in loop. i.e. x_{11}=200.
11. Subtract the x_{11} from (-) signed cells, and add to (+) signed cells in loop. i.e. $x_{11}=0, x_{22}=50, x_{21}=200, x_{12}=250$ and all other values remain same. Then u and v values for the modified allocated cells are in Table.2.0.10

	v_1=-3	v_2=1	v_3=0	v_4=-1
u_1=0	3	1	7	4
u_2=5	2	6	5	9
u_3=3	8	3	3	2

TABLE 2.0.10

12. Penalties are $P_{11}=-6, P_{13}=-7, P_{14}=-5, P_{24}=-5, P_{31}=-8, P_{32}=1$
13. P_{32} is positive. So select C_{32} and form a closed loop.
14. Select the least allocated value from the (-) signed cells in loop. i.e. x_{22}=50.
15. Subtract the x_{22} from (-) signed cells, and add to (+) signed cells in loop. i.e. $x_{22}=0, x_{33}=200, x_{23}=150, x_{32}=50$ and all other values remain same. Then u and v values for the modified allocated cells are in Table.2.0.11

	v_1=-2	v_2=1	v_3=1	v_4=0
u_1=0	3	1	7	4
u_2=4	2	6	5	9
u_3=2	8	3	3	2

TABLE 2.0.11

16. Penalties are $P_{11}=-5, P_{13}=-6, P_{14}=-4, P_{22}=-1, P_{24}=-5, P_{31}=-8$ and

$$X=\begin{bmatrix}0 & 250 & 0 & 0\\ 200 & 0 & 150 & 0\\ 0 & 50 & 250 & 150\end{bmatrix}$$

17. All penalties are lesser than zero, so optimality achieved. Then the optimal solution by using eq.(1.1) is = 2450 rupees.

3 Conversion to LPP

Problem 3.1. Find the optimal solution for the transportation problem in Problem.1.1 using LPP.

Solution:

1. The objective function is

$$f=\sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij}C_{ij}$$

2. Make the constraints
$3x_{11}+1x_{12}+7x_{13}+4x_{14}\le 250$
$2x_{21}+6x_{22}+5x_{23}+9x_{24}\le 350$
$8x_{31}+3x_{32}+3x_{33}+2x_{34}\le 400$
$-3x_{11}-2x_{21}-8x_{31}\le -200$
$-1x_{12}-6x_{22}-3x_{32}\le -300$
$-7x_{13}-5x_{23}-3x_{33}\le -350$
$-4x_{14}-9x_{24}-2x_{34}\le -150$
$x_{11},x_{12},x_{13},x_{14},x_{21},x_{22},x_{23},x_{24},x_{31},x_{32},x_{33},x_{34}\ge 0$
3. Write a python code to minimize f.

```
from cvxopt import matrix ,
    solvers

A = matrix
    ([[1.0,1.0,1.0,1.0,0.0,0.0,
 0.0,0.0,0.0,0.0,0.0,0.0],
[0.0,0.0,0.0,0.0,1.0,1.0,
1.0,1.0,0.0,0.0,0.0,0.0],
[0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,1.0,1.0,1.0,1.0],
[-1.0,0.0,0.0,0.0,-1.0,0.0,
0.0,0.0,-1.0,0.0,0.0,0.0],
[0.0,-1.0,0.0,0.0,0.0,-1.0,
0.0,0.0,0.0,-1.0,0.0,0.0],
[0.0,0.0,-1.0,0.0,0.0,0.0,
-1.0,0.0,0.0,0.0,-1.0,0.0],
[0.0,0.0,0.0,-1.0,0.0,0.0,
0.0,-1.0,0.0,0.0,0.0,-1.0],
[-1.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0],
[0.0,-1.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0],
[0.0,0.0,-1.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0],
[0.0,0.0,0.0,-1.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0],
[0.0,0.0,0.0,0.0,-1.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0],
[0.0,0.0,0.0,0.0,0.0,-1.0,
0.0,0.0,0.0,0.0,0.0,0.0],
[0.0,0.0,0.0,0.0,0.0,0.0,
-1.0,0.0,0.0,0.0,0.0,0.0],
[0.0,0.0,0.0,0.0,0.0,0.0,
0.0,-1.0,0.0,0.0,0.0,0.0],
```

```
[0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,-1.0,0.0,0.0,0.0],
[0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,-1.0,0.0,0.0],
[0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,-1.0,0.0],
[0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,-1.0]])
b = matrix([250.0, 350.0, 400.0,
    -200.0, -300.0, -350.0,
   -150.0,
   0,0,0,0,0,0,0,0,0,0,0,0])

c=matrix([3.0, 1.0, 7.0, 4.0,
   2.0, 6.0, 5.0, 9.0, 8.0, 3.0,
   3.0, 2.0])

sol = solvers.sdp(c, A.T, b)
print(sol['x'])
```

Problem 3.2. Verify the solution with MODI method.

4 Exercises

Problem 4.1. A transportation problem for which the costs, origin and availabilities, destination and requirements are given in TABLE 4.1.1

Check whether the following basic feasible solution

	D_1	D_2	D_3	
Q_1	2	1	2	40
Q_2	9	4	7	60
Q_3	1	2	9	10
	40	50	20	

TABLE 4.1.1

$$x_{11} = 20, x_{13} = 20, x_{21} = 10, x_{22} = 50$$
$$x_{33} = 10 \text{ and } x_{12} = x_{23} = x_{32} = x_{33} = 0$$

is optimal. If not, find an optimal solution.

Problem 4.2. The following TABLE 4.2.1 shows the information on the availability of supply to each warehouse, the requirement of each market and unit of transportation cost (in rupees) from each warehouse(W) to each market(M). The present transportation schedule is as follows:

		M				
		M_1	M_2	M_3	M_4	Supply
W	W_1	6	3	5	4	22
	W_2	5	9	2	7	15
	W_3	5	7	8	6	8
Requirement		7	12	17	9	

TABLE 4.2.1

W_1 to M_2: 12 units; W_1 to M_3: 1 unit; W_1 to M_4: 9 units; W_2 to M_3: 15 units; W_3 to M_1: 7 units and W_3 to M_3: 1 unit. Then the minimum total transportation cost (in rupees) is